ON THE IMPLEMENTATION OF NARROWBAND WAVE DIGITAL FILTERS

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In narrowband wave digital filters certain adaptor coefficients can take on extremely small values, which are impractical for a filter implementation based on fixed-point arithmetic. As it is described in this paper, the necessary wordlength for these adaptor coefficients can substantially be reduced by the introduction of additional elementary adaptor/one-port combinations. By this method, the number of distinct coefficient values is unchanged as compared to the original filter. However, the multiplication count is increased. An illustrative example is given.

1. STATEMENT OF THE PROBLEM

The implementation of narrowband digital filters (such as notch or enhancement filters) generally requires long wordlengths for coefficient and data representation. This is due to the fact that the poles and zeros of the transfer functions of these filters are very close to the z-plane unit circle /1.13/. Consequently, if transfer functions of this kind are, for instance, to be implemented on dedicated signal processor hardware with fixed-point single precision arithmetic, filter realization schemes requiring appropriate short wordlengths must be found. (Refer, e.g., to the NEC-signal processor /2/; 16 bit data bus, 1) bit coefficient ROM wordlength.) A common solution to this problem is to apply filter algorithms with a nonminimum number of multiplications /3,4/.

The same problem arises with narrowband wave digital filters (WDF /5,6/). In this case certain adaptor coefficients /7/ are very close to zero (or unity). In compliance with the above outlined solution, it is expected that the wordlengths in WDFs can be shortened by the introduction of additional circuitry. One way to arrive at this goal is shown in this contribution with emphasis on the reduction of the necessary coefficient wordlength w_m. For the connection between coefficient sensitivity and roundoff noise refer to /6.8/.

2. INTRODUCTION OF ADDITIONAL CIRCUITRY IN WDFs

Consider, for instance, the general wave digital ladder filter of Fig. 1. Here, the three-port adaptors A_m, where \{m\epsilon N_0\} and series or parallel adaptors /7/, respectively. The only general adaptor (A_m) is, most efficiently, placed in the centre of the main adaptor chain. The blocks E_m(z) are assumed to represent pseudolossless one-ports which, in the case of certain lowpass or highpass transfer functions, degenerate to simple delays with or without inverters /5,9/, respectively.

Let subsequently, for instance, the m-th adaptor of Fig. 1 be a (not necessarily
The equivalence expressed in Fig. 2 can be understood, if we decompose the lossless (reactive) one-port of the reference filter associated with the WDF-structures of Figs. 1 and 2 /5,9/, which corresponds to $E_m(z)$ into a sum of $K$ reactances according to

$$Z_m = \sum_{k=0}^{K-1} Z_{mk} = \sum_{k=0}^{K-1} \beta_{mk} Z_m$$

where obviously

$$\sum_{k=0}^{K-1} \beta_{mk} = 1$$

From (1) immediately follows that, in order to derive $Z_{mk}$ from $Z_m$, each inductance of $Z_m$ has to be multiplied by $\beta_{mk}$ and each capacitance of $Z_m$ has to be multiplied by $1/\beta_{mk}$. This is easily verified by expanding the reactance function $Z_m$ in partial fractions

$$\beta_{mk} Z_m = \beta_{mk} \left( \delta_{0} \left( \frac{1}{\psi_{c0}} \right) + \sum_{i=1}^{L} \frac{1}{\psi_{ci}} \right)$$

$$= \left( \frac{G_{mk}}{\psi_{c0}} + \frac{R_{mk}}{\psi_{c0}} \right) \beta_{mk} \sum_{i=1}^{L} \left[ \frac{G_{ii}}{\beta_{mk}} \right]$$

where $\delta_{0}$ denotes the structure of $Z_m$ is retained in $Z_{mk}$ for $0 \leq k \leq K-1$. Thus, every port resistance (conductance) of the associated WDF representation of $Z_{mk}$ corresponding to $R_{mk}(z)$ is weighted by $\beta_{mk} (1/\beta_{mk})$. Since the coefficients of N-port adaptors are exclusively determined by ratios of resistances or conductances /7/, respectively, we have

$$E_{mk}[z] = E_m[z]$$

as it is depicted in Fig. 2.

If we require for all adaptor coefficients $R_{mk}$ of the equivalence according to Fig.2

$$\min_{0 \leq k \leq K} \left\{ \alpha_{mk} \right\} = \max \left\{ \alpha_{mk} \right\}$$

in compliance with the outlined design goal, we obtain after a tedious but straightforward calculation for $0 \leq k \leq K-1$

$$k \alpha_{mk} = \alpha_{mk} = \sqrt{ \frac{2R_{mk}}{R_{mk}^2 + R_m^2} }$$

and, due to (1b) /7/

$$\alpha_{mk} = \alpha_m$$

These results apply to 3-port parallel adaptors accordingly. Furthermore, the generalization to N-port adaptors (N>3) is straightforward, since any N-port adaptor is realizable by elementary adaptors /7/.

In summary, when introducing K-1 additional adapter/one-port combinations in front of any (the n-th) independent port of a 3-port adaptor $A_m$, as shown in Fig. 2, we observe:

- Only the adaptor coefficient $G_{mn}$ of $A_{mk}$ related to this very port is influenced, i.e., increased according to (5).
- Each coefficient value of any additional adapter giving rise to a (hardware- or software-) multiplication can be made identical to $G_{mn}$ according to (5a).
- Each extra adaptor is reflection-free /7/, as it is necessary for realizability /7/.
- Each one-port of the equivalent representation is identical to that of the original filter according to (3).

Thus, despite the fact that the multiplication count is increased as compared to the original filter, the number of different coefficient values is exactly retained. Finally, it should be noticed that the coefficients assigned to the dependent ports of the adaptors $A_{mk}$, where $0 \leq k \leq K$, are also modified by the outlined procedure. However, those coefficients are eliminated from the filter algorithm /7,9/.

3. EXAMPLE

As an example we take the (symmetric) Chebyshev lowpass filter T0315 /10/. The original (canonical) wave digital filter with a normalized passband edge frequency of $f_m = 0.0004$ derived from the reference filter is depicted in the first row of Table 1 together with the corresponding adaptor coefficients. Its implementation requires a coefficient wordlength of about 22 bits, since $Q_2$ is extremely small. First, if on either side of the adaptor $A_2$ one adaptor/delay combination is introduced (Table 1, second row), an increased coefficient value $Q_2$ is obtained being in the same order of magnitude as $Q_1$. Hence, the coefficient wordlength can be shortened by about 7 bits. Next, four extra coefficient bits can be saved with four or six supplementary adaptor/delay combinations, as it is shown in the third and fourth row of Table 1, respectively. The associated attenuation responses are depicted in Fig. 3.
Table 1

Table 1  Example

<table>
<thead>
<tr>
<th>Implemented circuit of Chebyshev lowpass T0315 with $f_p = 0.0004$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First coefficient</td>
</tr>
<tr>
<td>$\alpha_1 = 1.220 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\alpha_1 = 1.220 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\alpha_1 = 3.405 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\alpha_1 = 4.043 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Obviously, the efficiency of the described method is most powerful in the first step.

4. CONCLUSION

It has been shown that the coefficient sensitivity of (narrowband) wave digital filters (WDF) implemented with fixed-point arithmetic can substantially be reduced by the introduction of supplementary adaptor/one-port combinations. Due to the interrelationship of roundoff noise and coefficient sensitivity in digital filters /6,8/, the roundoff noise (or the additional internal wordlength /11/) of WDFs with extra circuitry can be expected to be considerably lower than that of the associated original filter. Thus, aiming at an efficient (signal processor) implementation of (narrowband) WDFs taking into account signal-to-noise constraints, a trade-off between (coefficient and data) wordlengths and multiplier/adder count can be carried out (cf. Table 1). Moreover, since the number of different coefficient values can be kept independent of the amount of additional circuitry, the outlined method lends itself to the implementation of tunable WDFs with arbitrary passband width variations /12/.

REFERENCES