Joint Oversampling SBC-FDFMUX Filter Bank for $M$ Useful Channels Based on Modified DFT Filter Banks with Nearly Perfect Reconstruction

Mohammed N. Abdulazim, and Heinz G. Göckler

Abstract — Emerging digital on-board processors for communication satellites concurrently ask for both tasks of the cascade of a frequency demultiplexer and a frequency multiplexer: (i) mere de- and remultiplexing of an FDM signal (FDFMUX functionality) and (ii) subband coding of one signal (SBC functionality). In this paper, the recently proposed oversampling FIR SBC-FDFMUX filter bank for two useful channels, which efficiently combines both tasks, is extended for higher channel numbers using Modified DFT filter banks. Moreover, a design example for the extended SBC-FDFMUX filter bank is given.

Keywords — Digital Filter Banks, Modified DFT SBC-FDFMUX Filter Bank, Flexible On-Board Demultiplexing.

I. INTRODUCTION

The cascade of a frequency demultiplexing (FDMUX) and a frequency multiplexing filter bank (FMUX) is usually applied either for mere de- and remultiplexing of an uniform FDM signal (so called FDFMUX filter bank [1]) or for subband coding of one signal (SBC filter bank [2]) with (nearly) perfect reconstruction ((N)PR). Newly emerging digital on-board processor systems for communication satellites concurrently ask for both functionalities within one filter bank cascade. Firstly, since they allow for flexibly allocating different bandwidths to different users, if incoming FDM sub signals are non-uniform, i.e. they consist of channels of different bandwidths. Secondly, these on-board processors use on-board switching for variable channel to beam allocation and, therefore, always channelise the incoming FDM signal into single elementary granules of equal bandwidth resulting in the disintegration of channels having a higher bandwidth. The latter fact calls for the perfect reconstruction of these disintegrated channels.

The filter requirements for an FDFMUX and an SBC filter bank, respectively, obviously exclude each other: The efficiency of FDFMUX filter banks is based on the use of $M$th band filters, where roughly every $M$th coefficient is zero. On the other hand, SBC filter banks usually require power complementary filters for the FMUX as well as for the FMUX in order to exhibit the (N)PR property with a sufficiently flat filter passband characteristic [2], [3].

Recently [1], the oversampling FIR SBC-FDFMUX filter bank for two useful channels was presented combining the FDFMUX and the SBC functionality in a novel joint and yet efficient filter bank approach. It is derived from the FDMUX Universal Directional Filter Cell (UNDIFICE) designed for a decimation factor of $M=2$ and $I=4$ channels [4], from which only two are used: This 4-channel UNDIFICE is split into two 2-channel FDMUX filter banks, and each FDMUX is complemented with a succeeding 2-channel FMUX to form two SBC filter banks. In [5], design options for an NPR as well as a PR SBC-FDFMUX filter bank for two useful channels were investigated and compared with each other.

In [6], the SBC-FDFMUX filter bank was extended to more than two channels using the tree-structured filter bank approach. In addition, a third task was specified for the SBC-FDFMUX filter bank: The combination of the SBC and the FDFMUX functionalities in the same filter bank, a task emerging only in conjunction with more than two usable channels.

In this paper, we develop and investigate an oversampling SBC-FDFMUX filter bank for $M$ useful channels. In contrast to [6], we restrict ourselves to the FDFMUX and the SBC functionalities, i.e. the SBC-FDFMUX filter bank is either applied as an FDFMUX or as an SBC filter bank. The extension to $M$ channels is based on Modified DFT (MDFT) filter banks [7] with nearly power complementary square root raised cosine filters leading to NPR filter banks. The application of these closed-form linear phase filters avoids any form of spectral factorisation for filter bank design.

II. ADOPTED APPROACH TO SBC-FDFMUX FILTER BANKS WITH $M$ USEFUL CHANNELS

In contrast to [1], we start the development of the oversampling SBC-FDFMUX filter bank for $M$ useful channels from an oversampling SBC filter bank. As shown in [1], the $I=4$-channel SBC-FDFMUX filter bank for two useful channels consists of two critically sampling SBC filter banks. Hence, in order to obtain an $I=2M$-channel SBC-FDFMUX filter bank with $M > 2$, we split one...
oversampling SBC filter bank with \( I=2M \) channels into two critically sampling \( K \)-channel SBC filter banks with a decimation factor of \( M=K \).

The resulting (parallel) structure of this SBC-FDFMUX filter bank is shown in Fig. 1, where the even-numbered channels (index \( 2q \) with \( q = 0, \ldots, K-1 \)) belong to one SBC filter bank and the odd-numbered channels (index \( 2q+1 \)) to the other one. Additionally, Fig. 1 shows a potential bandlimiting block, which is necessary to allow for well-defined interface spectra before remultiplexing in case of the FDFMUX functionality.

\[
X_s(z_i) = \sum_{q=0}^{K-1} \hat{X}_q(z_i),
\]

where \( \hat{X}_q(z_i) \) are the bandlimited elementary granules of the FDMUX input signal after demultiplexing.

Fig. 2c shows the analysis filter functions for the FDFMUX functionality, which result from combining the filter functions of Fig. 2a and Fig. 2b, in compliance with the structure in Fig 1. Similarly to the 4-channel FDMUX UNDIFICE of [4], the \( l \)-channel FDMUX of the SBC-FDFMUX filter bank exploits the oversampling by a factor of two to allow for filter transition bands as wide as one filter bank channel, leading to a high efficiency.

\[
\hat{X}_q(z_i) = \sum_{i=0}^{M-1} \hat{X}_i(z_i)\delta_{i q},
\]

where \( \hat{X}_i(z_i) \) are an almost or exact perfect reconstruction of the input signal \( X(z_i) \).

In the following, some major characteristics of critically sampling MDFT filter banks are recalled before applying these filter banks to the SBC-FDFMUX approach exhibiting the SBC as well as the FDFMUX functionality.

III. MODIFIED DFT SBC FILTER BANKS

The Modified DFT SBC filter bank belongs to the family of complex modulated DFT SBC filter banks, where a real prototype lowpass filter is decomposed in its poly-phase components and all channel filter functions are obtained by an uniform complex modulation that is efficiently realised by the IFFT/FFT algorithm [2], [3].

As far as critically sampling NPR MDFT SBC filter banks \( (K=M) \) are concerned, the channel filter functions have to meet the Pseudo QMF (PQMF) requirements [7]:

i) The transfer functions of two adjacent channels must be approximately power complementary in the range between their centre frequencies, and the overall distortion function of the filter bank should be sufficiently close to a mere delay function.

ii) Furthermore, the main aliasing spectra directly neighbouring the useful spectra have to be compensated by proper methods.

iii) Finally, all other aliasing spectra have to be eliminated by a sufficiently high stop-band attenuation of the prototype filter [7].

The first and third requirements specify the design of the prototype filter, while the second requirement aims at, for example, structural modifications [8].

In the following, for convenience, we retain the channel indexing of the two SBC filter banks of Fig. 1, even though each of them can be described separately.

A. Prototype Filter and Channel Filter Functions

In the scope of this paper, we use the closed-form prototype filter design for MDFT filter banks investigated in [9]. With the considered closed-form approach, the real prototype filter \( H_p(z) \) of a filter bank with \( K=M \) channels is derived from the truncated discrete-time linear phase square root raised cosine lowpass filter with the impulse response [7], [9]

\[
h_p(k) = \frac{4rk \cos[\pi k(1+r)/M] + M \sin[\pi k(1-r)/M]}{[1-(4rk/M)^2]rkm}
\]

and the roll-off factor \( 0 < r \leq 1 \). For \( k = 0 \) and
The input sampling rate is determined by
\[ f_s = I \cdot 12 \text{MHz} = 96 \text{MHz} \]
and the number of channels for the critically sampling SBC filter banks is given by
\[ K = M = 4. \]
For the SBC-FDFMUX filter bank design, the real prototype lowpass filter as well as the bandlimiting filter for the FDFMUX functionality have to be considered.

A. Prototype Filter Design

In [4], [5], prototype filters are designed with a minimum stopband attenuation of $a_s = 50\,\text{dB}$ that is also adopted in the following. The roll-off factor $r$ in (1) determines the width of the transition band. In order to have a preferable low order prototype filter, the maximum possible width of the transition band is chosen by setting $r = 0.5$, resulting in a passband edge at $f_p = f_i/(2I)$ and a stopband edge at $f_s = 3f_i/2I$ (cf. Fig. 2).

The resulting FIR prototype filter length is $N=139$. In accordance with (5) and (6), we set $n_h = n_G = (N-1)/2 = 69$ with $\eta_h = \eta_G = 0$. Fig. 5a depicts the logarithmic magnitude response of the prototype. The distortion function [3], [7] of the SBC-FDFMUX filter bank (even-numbered channels) is depicted in Fig. 5b exhibiting a maximum deviation from a mere delay function of about $0.0051\,\text{dB}$. 

![Fig. 5. a) Logarithmic magnitude frequency response of the prototype filter; b) Logarithmic distortion function of the SBC sub-filter bank (even-numbered channels).](image)

B. Design of Bandlimiting Filters

For the FDFMUX task, the bandlimiting of the 10 MHz granules is applied at the operational rate $f_{\text{op}} = f_i/M = 24\,\text{MHz}$. The channels spacing of 12MHz is exploited for filtering, resulting in a width of the transition band of 2MHz. The bandlimiting is efficiently realised using (complex) FIR halfband filters (CHBF) [3]. For a minimum stopband attenuation of 50dB, the filter length of $F_{\text{bl}}(z_o)$ is $N_{\text{bl}} = 31$, of which 14 coefficients are zero, resulting in an effective filter length of the CHBF of $N_{\text{eff}} = 18$. 

C. Computational Load

Each of the two critically sampling $K$-channel MDFT SBC filter banks of the SBC-FDFMUX filter bank is efficiently realised as a complex modulated DFT filter bank. The high potential of further reduction of the computational load $A$, i.e. the number of real multiplications per time unit (operations per second: Op/s), by exploiting the structural modification of MDFT filter banks, as investigated in [10], have not been considered so far.

The computational load of the presented SBC-FDFMUX filter bank is determined by the prototype filtering ($A_{\text{PTF}}$), the complex modulations with the initial phase ($A_{\text{IP}}$), the modulations for the odd channel allocation scheme ($A_{\text{OCA}}$). Note that the $K=4$-point IFFT/FFT is realised without any multiplication. All the above tasks are realised for each SBC filter bank of the SBC-FDFMUX filter bank twice: for its FDMUX and its FMUX, respectively. The potential bandlimiting ($A_{\text{BL}}$) applies only for the FDFMUX functionality, and only for the $J=2=4$ useful channels.

Table 1 gives a survey of the computational loads. Each complex multiplication is realised by 4 real multiplications.

**Table 1. Survey of the Computational Load for the $J=8$ Channel SBC-FDFMUX Filter Bank.**

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{PTF}}$</th>
<th>$f_{\text{op}}$ in MHz</th>
<th>Total in MOp/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>complex</td>
<td>139</td>
<td>4-139-48 = 26688</td>
</tr>
<tr>
<td>$A_{\text{OCA}}$</td>
<td>complex</td>
<td>$K=4$</td>
<td>4-4-48 = 768</td>
</tr>
<tr>
<td>$A_{\text{IP}}$</td>
<td>complex</td>
<td>$K=1-3$</td>
<td>4-3-48 = 576</td>
</tr>
<tr>
<td>$A_{\text{BL}}$</td>
<td>complex</td>
<td>18</td>
<td>4-18-24 = 1728</td>
</tr>
<tr>
<td>$A_{\text{FDMUX}}$</td>
<td></td>
<td></td>
<td>112128</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, critically sampling Modified DFT SBC filter banks with linear phase analysis and synthesis filters and the NPR property are used to realise a joint oversampling SBC-FDFMUX filter bank for more than 2 useful channels, which efficiently combines the SBC as well as the FDMUX functionality of an FDMUX/ FMUX cascade. Moreover, a design example is given. For future research, the high potential of more efficient realisations of MDFT filter banks will be considered for developing highly efficient MDFT SBC-FDFMUX filter banks.

**REFERENCES**


