MODEL-BASED SIMULATION OF A CLASS OF DIGITAL FDM-DEMULTIPLEXERS IN BEAMFORMING ENVIRONMENT

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Abstract

This paper reports on a model-based approach to simulation for a set of identical digital complex-domain FDM demultiplexers embedded in a beamforming network (BFN) for individual channel beamsteering [4,5]. The signal to overall distortion ratio is determined by simulation of one sensor path only, whereas the BFN superposition properties are modelled. Channel signals and aliasing distortions combining coherently in the BFN, and quantisation and clipping noise combining non-coherently, are separated by subsequent simulations. As an example, this simulation method is used to verify the design results reported in [5] for a 16-slot tree type FDM demultiplexer processing complex-valued signals in conjunction with oversampling by a factor of two.

1 Introduction

Future advanced satellites for mobile communications will increasingly employ digital signal processing techniques. These include on-board demultiplexing and remultiplexing, bulk traffic and individual channel routing, regeneration and remodulation techniques and, in conjunction with frequency re-use, digital beamforming achieved by multiple spot beam or array antennae [1-3]. A subsystem of a communications satellite typically encountered in a variety of applications (e.g. mobile satellite communications [4-6]) is depicted in Fig.1: In an appropriate intermediate frequency (IF) range the preprocessed FDM sensor output signals of an Ng-element array antenna are digitised (A/D), demultiplexed (FDM-DEMUX) to the frequency slot level (temporal filtering related to channel spacing B), and spatially filtered by a beamforming network (BFN weighting in conjunction with the subsequent combiner). In case of frequency re-use each frequency slot (index l ∈ [1, L]) may contain up to M > 1 spatially disjoint co-channels (index m ∈ [1, M]). Having access to single channels characterised by l, m or to groups of channels as desired, demodulation (if applicable), switching/routing, remodulation (if necessary) and remultiplexing (MUX) can be accomplished in any arbitrary format.

When looking for the implementation of the FDM-DEMUX quite a lot of demultiplexing methods are at choice (e.g. [7-9]) such as: slot individual filtering applying one or two step sample rate reduction; sharing filter functions with some or all slots (polyphase-FFT approach); multistage (binary) split-and-decimation methods. Note that the transition from real (single line) to complex signal representation (double line), necessary for beamforming, is most efficiently performed in conjunction with the demultiplexer function [9]. However, other approaches are conceivable, as indicated in Fig.1 (ASP, A/D).

In the case of digital beamforming, as it is assumed throughout subsequently (Fig.1), the narrowband conditions of beamforming are usually met. Hence, each BFN weighting block merely contains a single complex multiplier to control amplitude and phase of the associated sensor slot signal. However, the block diagram of Fig.1 is likewise valid for broadband beamforming, where the above complex multipliers are replaced by transversal filters for processing of complex-valued signals [10]. (In digital beamforming broadband conditions may prevail in conjunction with high speed digital signal processing [6] or with stringent array antenna pattern requirements.)

The design of a digital DEMUX-BFN system (shaded in Fig.1) usually starts with the analysis of all potential deteriorations such as spectral foldover (aliasing) and quantisation noise that are introduced by the system [9]. From this analysis and the specified performance requirements the various system parameters (filter stopband rejection and signal wordlengths) can be derived as detailed, for instance, for a tree-structured DEMUX in [5].

In a subsequent step the design results have to be verified by simulation that exactly considers finite-precision operations. Note that the simulation of an entirely digital system on a (general purpose) computer always represents a particular (but generally non-real time) implementation of the system. Furthermore, simulation is used as a tool for fine tuning (final optimisation) especially of those system parameters that are not or hardly accessible by mere analysis. Most importantly, one-to-one system simulation is needed as a reference for system or chip test in case of an application specific integrated circuit (ASIC) development.

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In this paper the one-to-one simulation of a set of digital FDM-DEMUXes is addressed, where these DEMUXes are part of a beamforming network (Fig.1). Obviously, all DEMUX blocks of Fig.1 are identical. Nevertheless, in a complete simulation of the DEMUX-BFN system all $N_F$ DEMUXes have to be simulated, since the input sequences to all DEMUXes are different due to sensor location dependent time of signal arrival and different sensor path noise received from space and produced by ASP and A/D. As a consequence, the inherently generated noise contributions of the various DEMUXes are obviously different.

A complete simulation as outlined would be extremely time-consuming, especially in the case of a large number of sensor elements ($N_F$) and/or a large number of frequency slots ($L$). Therefore a model-based simulation is proposed in this paper. In this approach a one-to-one simulation of only one sensor path is carried out comprising an A/D converter, one FDM-DEMUX, one BFN weighting block and the BFN combiner (adder); Fig.1. Thus, the FDM-DEMUX (eventually to be manufactured as an ASIC) is simulated exactly, whilst the overall DEMUX-BFN system is modelled as it is described in the subsequent sections.

In Section 2 the DEMUX-BFN model is defined, and the baseline of the system performance requirements is given. The model-based approach to simulation is introduced and detailed in Section 3. In Section 4 this approach to simulation is, as an example, applied to the DEMUX-BFN system as described in [5]. The design results of [5] are verified and additional outcomes and improvements obtained by simulation-based fine tuning are reported. Finally, in Section 5 some extensions of the described approach to simulation are discussed.

2 System Description

The FDM signals at the input ports of the FDM DEMUXes of the $N_F$ sensor paths consist of $L$ adjacent (frequency) slot signals each comprising up to $M$ superimposed co-channels (Fig.1). The spectra of these FDM signals have, in general, different phases but identical magnitudes. The latter deterministically only holds when assuming noise free conditions.

The FDM DEMUXes, a set of filter banks, separate each of the $N_F$ FDM signals into $L$ individually accessible slot signals. The combination of all $N_F$ slot $l$ signals $(l = 1,2,\ldots,L)$ via the BFN weighting blocks for amplitude and phase steering optimally yields the desired channel $l$, $m$ and nulls all other $M - 1$ co-channels [4].

The quality of signals while being processed in the digital DEMUX-BFN system including A/D suffers from three different types of distortions [5,9]:

i) Aliasing (spectral foldover) caused by decimation in conjunction with non-ideal digital filtering in DEMUX.

ii) Input quantisation and re-quantisation (i.e. shortening the signal wordlength at the rear end by dropping least significant bits in a suitable manner) at various points all over the DEMUX-BFN system.

iii) Clipping, i.e. constraining a signal $s(k)$ not to exceed an overflow threshold $C$. Note that clipping is generally connected with (re-)quantisation, but only occurs at some quantisation points. Subsequently saturation
clipping is assumed characterised by:
\[
\begin{align*}
  s(k) &\equiv C & V_s(k) &\geq C \\
  s(k) &\equiv -C & V_s(k) &\leq -C
\end{align*}
\]
(1)

Applying the saturation level \( C \) the quantisation step size is defined:
\[
q = C \cdot 2^{-w+1}
\]
(2)
where \( w \) represents the signal wordlength including sign.

2.1 DEMUX-BFN Model

The DEMUX-BFN system model is introduced in compliance with \([5,11,12]\).

Channels (Desired Signals)

All \( L \cdot M \) (Desired Signals)

All \( L \cdot M \) channels are considered **uncorrelated** with each other. As a consequence of appropriate BFN weighting all \( N_E \) sensor path signals are multiply combined such that always \( N_E \) channels \( l,m \) appear coherently superimposed at the \( L \cdot M \) BFN output ports, while the undesired co-channels are nulled. In order to reject these \( M-1 \) co-channels
\[
N_E \geq M
\]
(3)
sensors are necessary. Hence, the overall input signal power to each of the \( N_E \) DEMUXes is given by:
\[
P = \frac{1}{N_E} \sum_{l=1}^{L} 1 \sum_{m=1}^{M} P_{l,m}
\]
(4)
where \( P_{l,m} \) represents the power of channel \( l,m \) at DEMUX-BFN system output port and \( a_l > 1 \) a (possibly slot dependent) scaling factor \([9]\).

Aliasing

Basically, the \( N_E \) sensor path aliasing contributions are combined **coherently** by the BFN. However, the spatial locations (directions of arrival: DOA) of aliasing signals are different both from that of the desired channel \( l,m \) and from those of the other \( M-1 \) nulled co-channels. To cope with these features of the BFN, a **beamforming coherence factor** \( m_a \) is introduced where
\[
0 \leq m_a = M_a/M < 1
\]
(5)
is anticipated subsequently. \( M_a \approx 0 \) is only possible with an antenna beam pattern exhibiting at least \( M \) nulls (\( N_E > M \)). \( M_a < M \) is valid since the absolute maximum of the beam pattern is assumed to be exactly directed to the DOA of the desired channel.

Quantisation and Clipping

The various noise sequences introduced in all \( N_E \) sensor paths by quantising and clipping operations are considered **omnidirectional and uncorrelated** with each other and with the associated generating signals. However, the overall power levels of the \( N_E \) sensor path noise contributions are assumed to be equal. Furthermore, quantisation and clipping noise is considered white.

2.2 System Performance Requirements

The performance measure adopted for the DEMUX-BFN system is the **signal to overall distortion ratio** (SDR) at system output \([5,11]\):
\[
SDR = 10 \log_{10} R
\]
(6)
where
\[
R = \frac{P_{l,m}}{A + N}
\]
(7)
tactfully assuming undistorted FDM input signals to the DEMUX-BFN system \((S/N_{in} \rightarrow \infty)\). In (7) \( A \) represents the total power of the aliasing contributions and \( N \) the overall sum of the quantisation and clipping noise power levels. All these distortions \((A + N)\) are exclusively produced by the DEMUX-BFN system.

By incorporating the DEMUX-BFN system model developed in the preceding subsection eq.(7) is rewritten as follows:
\[
R = \frac{N_E^2 P_{l,m}}{N_E^2 m_a A_l + N_E N_l + N_{out}}
\]
(8)
with obvious relationship with the numerator and denominator terms of (7). In (8) the quantities \( P_{l,m}, A_l \) and \( N_l \) are related to the slot level at the input ports of the BFN combiner (cf.Fig.1). \( N_{out} \) refers to re-quantisation at DEMUX-BFN output. Note that, for convenience, the power levels of all \( M \) co-channels are assumed identical.

By means of simulation the power quantities \( P_{l,m}, A_l \) and \( N_l \) in conjunction with \( N_{out} \) can be determined separately, to be shown next, and combined in compliance with the model eq.(8) to yield the performance of the DEMUX-BFN system.

3 Approach to Simulation

The time domain approach to the model-based simulation is depicted in Fig.2 in conjunction with the spectra of the respective input signals. Obviously **three different simulations of one sensor path** of the overall DEMUX-BFN system have to be carried out to determine the system performance SDR as defined by (6)-(8). For clarity of presentation the simulation method is described for the class of FDM DEMUXes with oversampled complex valued input and output signals, for instance according to \([5,9]\). However, this simulation method is not restricted to this class of digital multirate filters (cf. also \([13]\)).

In all three simulations the system is fed by a periodic noise-like FDM signal with a well-defined spectral distribution, which is specific for each case. This is accomplished by transforming the respective spectra \( S_\lambda(\mu) \), \( \lambda \in \{a,b,c\} \) each specified as a discrete line spectrum, by means of the inverse discrete (fast) Fourier transform (IFFT) to the time domain \([12,14]\):
\[
S_\lambda(k) = IFFT \left[ S_\lambda(\mu)e^{j\phi(\mu)} \right], \quad \lambda \in \{a,b,c\}
\]
(9)
where $\mu \in \{0, N_{FFT} - 1\}$ stands for the equidistant frequency points of the spectral lines, and underlining indicates complex-valued quantities, signals or spectra thereof. Subsequently the phases $\varphi_\lambda(\mu)$ of (9) are chosen in a random manner uniformly distributed in $[-\pi, \pi)$. Hence the periodic sequences $s_\lambda(k), \lambda \in \{a, b, c\}$, exhibit an approximately normal distribution [1]. (Note that ergodic processes have tacitly been assumed [12].) Using this fact in conjunction with a peak factor $p_f$ [5], the power levels of the noise-like FDM signals can be defined as appropriate for the three simulations.

3.1 Quantisation and Clipping Noise

The simulation procedure for the isolated determination of quantisation and clipping noise is depicted in Fig.2a: The DEMUX as well as BFN weighting and summation is implemented in finite precision arithmetic compliant with design results.

The system is fed by a signal $s_\lambda(k)\rightarrow s_\lambda(\mu)$ where the considered slot $l$ as well as every other slot is idle. The remaining (half of all) slots are equally loaded with

$$|s_\lambda(\mu)| = S_\lambda = \text{const} \quad (10)$$

The choice of this particular type of FDM input spectrum

i) prevents the considered slot $l$ from being aliased by any of the loaded slot signals,

ii) makes sure that all quantisation (Q) and clipping (saturation level $C_v$) noise sources inherent in the DEMUX-BFN system are adequately activated if the overall input power is set appropriately.

In order to determine the power (variance) of the complex-valued DEMUX input signal, assuming a Gaussian process both for its real ($r$) and its imaginary ($i$) component with

$$\sigma_r^2 = \sigma_i^2 = 2\sigma^2 \quad (11)$$

a relationship to the spectral density (10) has to be established. To this end a common peak factor is introduced by

$$p_f\sigma = C_1 = 1 \quad (12)$$

where subsequently, for convenience, the saturation threshold $C_1$ is set to unity [5].

The power of signal $s_\lambda(k)$ is given by

$$\sigma^2 = \frac{1}{N_{FFT}} \sum_{k=0}^{N_{FFT}-1} |s_\lambda(k)|^2$$

$$= \frac{1}{N_{FFT}} \left[ \frac{1}{N_{FFT}} \sum_{\mu=0}^{N_{FFT}-1} |S_\lambda(\mu)|^2 \right] \quad (13)$$

where use is made of the Parseval identity of the discrete Fourier transform [12]. With (10) and the half-loaded spectrum $|S_\lambda|$, depicted in Fig.2a, the square bracket of (13) turns out to equal $S_\lambda^2/2$. Hence the combination

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of (13) with (11) and (12) establishes the desired functional relationship
\[ S_a = \frac{2}{P_f} \sqrt{N_{F_{FFT}}} \] (14)

The complex-valued output sequence of BFN weighting (containing \( N_{f} \)) is multiplied by \( \sqrt{N_F} \) according to the system model of Section 2 and re-quantised behind the BFN combiner. Since oversampling is assumed up to system output, a final band limitation filter BLF (which could also be part of the system to be simulated) is necessary to reject all spectral components lying not within slot \( l \) before power measurement according to (13). Clearly, power measurement only starts when the DEMUX-BFN system has reached steady state, which is unambiguously defined for nonrecursive systems.

### 3.2 Aliasing Contributions

The simulation procedure for aliasing contributions is depicted in Fig.2b. Since aliasing must not contain any quantisation noise (cf. system model of Section 2), this system simulation is performed in floating-point arithmetic (approximately infinite precision). The phase shifting of BFN (multiplication by \( e^{j\phi} \)) could be discarded since it does not affect power measurement. (Note that multiplication by \( e^{j\phi} \) is necessary in simulation a in order to activate the BFN weighting re-quantisation noise sources.)

In order to measure the power aliased to slot \( l \), similarly to simulation a slot \( l \) of the FDM input spectrum has to be idle, whereas all slots contributing to aliasing must be loaded as specified. Since aliasing mechanisms are different for different DEMUX methods, all slots except for slot \( l \) are loaded in order not to be restrictive.

While \( S_a \) was determined to correctly activate all quantisation and clipping noise sources, \( [S_b(\mu)] \) and \( [S_c(\mu)] \) (3rd simulation: Fig.2c) have to comply with system specifications, the conditions defining SDR (6)-(8). For convenience, again
\[ [S_b(\mu)] = S_b = S_c = const. \] (15)

is subsequently assumed for all loaded slots, as shown in Fig.2b.c. (Other choices are straightforward.) Hence, the nominal case with all channels equally loaded [11] is characterised by
\[ S_b = S_c = \frac{1}{P_f} \sqrt{2N_{F_{FFT}}} = S_a/\sqrt{2} \] (16)
derived by correspondingly applying (11)-(14).

The complex-valued floating-point DEMUX-BFN output sequence (containing \( A_l \)) is weighted by \( N_E \cdot \sqrt{m_a} \) in compliance with the adopted system model. Again BLF operation and power measurement is performed as described for simulation a.

### 3.3 Channel Signal

In a final simulation (Fig.2c) the system output power of an undistorted channel has to be determined by feeding the DEMUX with a single slot \( l \) signal at nominal level defined by \( S_c \) according to (16). This measurement is necessary for reference purposes to cope with scaling and the (slightly slot dependent) in-band frequency response of DEMUX.

The complex-valued DEMUX-BFN output sequence (containing \( P_l \)) is weighted by \( N_E/\sqrt{M} \) according to Section 2. Its squared magnitude is averaged to yield \( P_{lm} \). It should be noted that band limitation to \( B \) by BLF, being superfluous in this simulation, is introduced to have identical post-system signal processing in all three simulations.

### 4 Example

The DEMUX-BFN system, the features and design of which are reported in [5], is adopted as an application example for the described simulation method. The FDM-DEMUX for processing of complex-valued signals is of the tree type structure: Each of the identical universal directional filter cells (UNDIFICIE as shown in Fig.3) splits the respective number of input channels into two groups of

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**Figure 3:** Universal directional filter cell (UNDIFICIE); \( e_p = \gamma h_p \) except for \( e_p = \gamma h_p/\sqrt{2} \); \( h_p \): real coefficients of halfband prototype channels each comprising half the original number of channels, while decimating the sampling rate by two. At any stage of processing all signals are oversampled by a factor.
of two, which is also valid for input and output sequences. Hence
\[
 f_{Si} = 2LB
\]
represents the DEMUX input sampling rate. For more details refer to [9,15].

Before reporting specific simulation results, the main intermediate steps of simulation \(a\) (for quantisation noise) are discussed for the above type of DEMUX with reference to the spectral representation of Fig.4 (8-slot DE
MUX calling for the cascade of three UNIFICE stages). To this end slot \(l = 3\) is considered throughout which, as every other slot, is idle at DEMUX input (Fig.4a). The numbers of the adjacent (unusable) slots are put in square brackets.

Figure 4: Spectral representation of simulation \(a\) for a tree type FDM DEMUX according to [5,9,15]

After stage 1 filtering by \(H\) (Fig.4a) and decimation by two the aliased spectrum of Fig.4b results. Obviously slot \(l = 3\) is still idle, as anticipated, except for the white noise floor (not indicated in Fig.4) that is due to stage 1 inherent re-quantisation and clipping. Furthermore, all other originally idle slots remain idle. The power of the loaded slots is doubled relative to their original power level as a consequence of UNIFICE inherent scaling by \(\gamma = \sqrt{2}\) [5]. (Note ordinate scaling of Fig.4.) This is evident for slots 2 and 4 aliased by a negligible contribution that was subjected to UNIFICE stopband rejection. However, slots 6 and 8 are partially attenuated by \(H\). The same happens to the aliasing slots 14 and 16, respectively, in a complementary manner, since UNIFICE is a halfband (or Nyquist)
filter [9,15]. Thus, the overall input power to the stage 2 UNIFICEs (Fig.4b) is the same as that of stage 1.

Signal processing in the subsequent stages is performed similarly. The only slight difference occurs in stage 2 where slot \(l = 3\) is shifted to a different frequency position (Fig.4b,c). Note that at DEMUX output the power level is still the same as that at DEMUX input (Fig.4d).

Hence, the BFN clipping and quantisation noise sources are correctly activated (cf. Fig.2a). Finally, in Fig.4d band limitation BFL is shown to exclusively capture the quantisation and clipping noise floor of slot \(l = 3\).

The DEMUX-BFN system of [5,11] is characterised by the following parameters:

- \(N_E = 10\) sensors of array antenna
- \(M = 3\) frequency re-use factor
- \(m_a = 2\) BFN coherence factor for aliasing
- \(L = 16\) number of DEMUX frequency slots
- UNIFICE: linear-phase complex FIR halfband filter
- \(n_f = 11\) filter length of UNIFICE
- \(a_s \geq 48.56\) dB stopband rejection of UNIFICE
- \(\gamma = \sqrt{2}\) scaling factor of UNIFICE
- All \(L\) usable and all \(L\) adjacent slots are equally loaded (nominal case)
- \(w = w_P = 8\) bit DEMUX and UNIFICE I/O signal wordlength
- \(w_B = 10\) bit BFN signal wordlength.
- \(SDR \geq 33\) dB signal-to-distortion ratio

A first simulation result obtained with the above DEMUX-BFN system is shown in Fig.5 in connection with the system performance predicted by design for an UNIFICE intracell signal wordlength of \(w_0 = 9\) bit and mathematical rounding (\(r\)). Simulation well confirms the design results for higher peak factors (low input power), where deterioration is governed by rear end re-quantisation. For clipping (lower peak factors), however, system analysis yields somewhat too pessimistic results. This is attributed to the fact that the assumptions of [5] for clipping noise modelling are exactly met at the DEMUX input ports only (i.e. a signal clipped once and re-processed is no longer normally distributed). In the range of maximum SDR the performance of slot 4 exceeds that of slot 1, since the various aliasing contributions of slot 4 are subjected to a higher average attenuation.

The second result (Fig.6) shows an application of the simulation tool for fine tuning of system parameters. As detailed in [5], multiplication is to be replaced by shift and add/subtract operations (canonical signed digit code representation of coefficients). Following this approach
5 Conclusion

The simulation of a finite arithmetic implementation of a class of digital FDM demultiplexers is described. The DEMUX is part of a beamforming network (BFN), where each sensor path has its own DEMUX. All these DEMUXes are identical. The simulation aims at determining the output signal to distortion ratio of the DEMUX-BFN system. In order to reduce CPU time consumption dramatically, the simulation of a one-to-one mapping of only one sensor path of the DEMUX-BFN system is carried out, whereas the beamforming environment is modelled. This model implies that both desired and (unwanted) aliasing signals of different sensor paths of the array antenna combine coherently, whereas quantisation noise contributions (including clipping) of different sensor paths are uncorrelated.

So far the incoming signals of the DEMUX-BFN system have always been anticipated as undistorted corresponding to \((S/N)_{in} = \infty\). Thus, the SDR according to (6)-(8) is obtained at system output. In the general case the overall signal to noise ratio at BFN output is given by:

\[
(S/N)_{out} = 10 \log_{10} \frac{1}{10^{-SDR/10} + \frac{1}{N_E}10^{-(S/N)_{in}/10}}
\]

where again all noise contributions due to (analogue) pre-processing in the sensor paths (cf. [5]) and due to omni-directional background noise received by the sensors are assumed uncorrelated. Obviously, the improvement of \((S/N)_{out}\) over \((S/N)_{in}\) achieved by the array antenna system approaches \(10 \log_{10} N_E\) on the dB-scale for \(SDR >> (S/N)_{in}\).

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