A Modular Approach to a Digital 60-Channel Transmultiplexer Using Directional Filters

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Abstract—A multistep transmultiplexer approach with a single-way modulation scheme applying minimum phase wave digital directional or transversal filters is proposed. Furthermore, the impact of signaling, pilots, and spectral shaping at the PCM end is investigated. The results of a filter design based on wave digital filters is given.

The main merits of this approach are, at a moderate level of computational complexity, its absolute stability, the low basic group delay, and its modularity. Due to modularity, the central unit (the TDM/FDM and FDM/TDM translators) is composed of only three different filter types requiring neither DFT, FFT, or associated processors nor modulators. As a consequence, the new approach is suitable for LSI/VLSI integration with the potential of efficient testability.

I. INTRODUCTION

In the past decade, several substantially different methods have been described for translating time-division multiplex (TDM) signal groups into frequency-division multiplex (FDM) group signals (and vice versa) using digital signal processing. For such systems (and all associated subunits), the expression “transmultiplexer” has been coined. The fundamental mode of operation of a 60-channel transmultiplexer is depicted in Fig. 1, where Fig. 1(a) is related to the TDM/FDM translation and Fig. 1(b) to the FDM/TDM translation. In its basic function, a transmultiplexer represents a multiple single-sideband modulator (SSB-MOD). For simplicity, subassemblies (such as those used for signaling and the generation and detection of pilots) have been omitted in Fig. 1.

At each of the two receivers R [Fig. 1(a)], a TDM signal arrives at a rate of 2.048 Mbits/s, corresponding to 30 PCM voiceband signals (PCM 30). Here, the two TDM signals are synchronized and scaled for level matching. Next, the interleaved TDM signals are converted to a parallel format, suitable for subsequent signal processing. Then the samples of the PCM signals, coded according to the A-law (µ-law) [1], are expanded in the block EX to form linear PCM signals. Finally, these linear signals are fed into a bank of digital single-sideband modulators, SSB-MOD, generating a digital FDM signal. From the latter, the associated continuous FDM signal is derived by means of a digital-to-analog converter D/A, followed by a filter FI for smoothing the staircase output signal of the D/A converter. In order to avoid an additional analog frequency translation necessitating one more modulator and filter, it is desirable that the 60-channel FDM signal at the input port of the filter FI directly be the FDM supergroup signal in the specified 312-552 kHz range. Then the post-D/A filtering is simply accomplished by a bandpass filter.

Performing the operations depicted in Fig. 1(a) in reverse order, the FDM/TDM translator of Fig. 1(b) results. The FDM/TDM translation can also be derived formally from the associated TDM/FDM translation by a transposition of its flow graph [2]. For this reason, the following investigations can, without loss of generality, be limited to the treatment of the TDM/FDM translation or, in particular, to the discussion of the digital generation of single-sideband signals by means of the subassembly SSB-MOD.

For the digital implementation of the multiple single-sideband modulator SSB-MOD, the central subassembly of a transmultiplexer, various solutions have been published in the past. The survey papers of Freeny [3], [4], Fettweis [5], and Scheuermann and Göckler [6] give a general view of the present state of the art. The following discussion is based on the subdivision of the transmultiplexer approaches into four classes [6]:

- bandpass filter-bank method
- low-pass filter-bank method
- Weaver-structure method
- multistage modulation method.

Furthermore, a distinction will be made between single-, two-, and multiple-way modulation schemes [5].

In order to motivate our choice (to be described in Sections II and III), we are going to investigate three essential features...
of digitally implemented transmultiplexers:

- stability, in particular under looped conditions
- absolute value of the group delay
- modularity.

We begin with the aspect of stability. As pointed out by Fettweis [5], stability problems can arise when using digital transmultiplexers, since they operate in the four-wire branch of an essentially two-wire transmission line (Fig. 2). Due to the four-wire/two-wire transitions (hybrids), transmultiplexers actually operate in a looped arrangement. Stability, however, must be guaranteed under any adverse condition, such as strongly unbalanced hybrids, which may occur in practice. This is particularly crucial, since parasitic oscillations will generally disturb not only the telephone channel under consideration, but also a multiplicity of, if not all, adjacent channels.

One way to avoid these parasitic oscillations under any practical circumstances is to use transversal (FIR) filters. Alternatively, wave digital filters (WDF) may be used when the exclusive utilization of FIR filters is inefficient. For these filter classes, potential stability under looped conditions has been proved by Meerköter and Fettweis [7]. They showed that only minor measures are required for guaranteeing absolute stability. Note that their proof is based on the implicit assumption of a single-way modulation scheme, meaning that each input signal to the TDM/FDM translator arrives at the output in a separate way. Furthermore, this stability theory can be extended to two-way modulation schemes using Hartley or Weaver modulators [8] for processing complex-valued signals. However, in order to assure absolute stability in this case, the required computational complexity is substantially higher than that of single-way modulation schemes. Initial attempts to apply this theory to transmultiplexing methods utilizing discrete Fourier, sine, cosine, or Hadamard processors, respectively, indicate that, as compared to a two-way modulation scheme, much additional circuitry would still be required for guaranteeing absolute stability (if it could be achieved at all) under looped conditions [5].

Following the arguments of Fettweis [5] rigorously, we are forced to drop the computationally most efficient transmultiplexer approaches such as the low-pass filter-bank method [9], the bandpass filter-bank method [10], and certain Weaver structure methods [11], [12]. As a consequence, for the subsequent discussion, essentially the multistep single-way approach of Fettweis [13] and the multistage two-way modulation scheme described by Tsuda et al. [14] remain. (Some Weaver-structure methods of lower efficiency [6] are disregarded.)

First, let us take a closer look at the highly modular transmultiplexing method of Tsuda et al. [14]. Here, an acceptable computational complexity (multiplication rate per channel: 760 000/s [6]) is achieved by using exclusively FIR halfband filters [15] from the second stage on. In contrast to its high degree of modularity, this approach exhibits, however, three essential drawbacks.

- A higher amount of additional hardware is required for this two-way modulation scheme to guarantee stability under looped conditions as compared to the approach of Fettweis [5], [7], [13].
- The extensive use of the computationally efficient FIR halfband filters with linear phase introduces a high basic group delay of about 2.6 ms [6], leaving only a small margin to the maximum allowable value of 3 ms [16].
- Due to its sampling frequency of 512 kHz at the FDM end, this approach requires a supplementary analog frequency conversion [6].

In contrast to [14], the transmultiplexer of Fettweis [13] is composed exclusively of minimum phase filters. Hence, the absolute group delay requirement of [16] can be met with a considerably greater margin. Furthermore, the FDM supergroup signal can directly be derived from the D/A converter output signal originally sampled with 576 kHz by using a bandpass filter for post-D/A conversion smoothing (cf. Fig. 1(a)). A significant disadvantage of this multistep approach based on a single-way modulation scheme is, however, its need for a somewhat greater number of different filter types and, in addition, a small number of modulators requiring only multiplications by +1, -1, and 0. As a consequence, this approach is substantially less modular than that of [14]. (Here, a high degree of modularity corresponds to a small number of different subunits or modules.) It must be noted that, in general, a digital system approach with an inherently low degree of modularity is expected to be inefficient with respect to system control and, in particular, to industrial development, production, and testing [5], [6]. Furthermore, it must be emphasized that a low degree of modularity usually prevents an LSI/VLSI system implementation, since the application of LSI/VLSI techniques for integration is, for economical reasons, generally limited to modules of which a great number is required.

In order to increase the degree of modularity of the transmultiplexing method of [13], an alternative multistage transmultiplexer approach using a single-way modulation scheme is proposed in this paper. This novel transmultiplexer is composed of digital directional filters such as WDF or FIR filters, and needs neither processors for orthogonal transforms nor modulators (except those used for spectral inversion). Note that the advantages of the transmultiplexer approach due to Fettweis [13] are preserved: stability even under looped conditions, moderate computational complexity, compliance with the CCITT recommendations [16] for absolute group delay leaving an ample margin, and the absence of any analog frequency conversion of the FDM signal remaining valid.
The organization of the paper is as follows. First, in Section II, the basic mode of operation of the proposed transmultiplexing method is described, taking into account signaling and the generation and detection of the pilots. Subsequently, in Section III, various approaches to an efficient implementation of the individual modules and subassemblies of the transmultiplexer are discussed. Finally, in Section IV, a filter design example based on the unique application of wave digital directional filters is reported, and the associated filter responses are given.

II. DESCRIPTION OF THE PROPOSED SYSTEM APPROACH

In Fig. 3, the multiple single-sideband modulator block SSB-MOD of Fig. 1(a) is shown with all additional subassemblies necessary for practical use in the telecommunication network: the spectral shaping of the incoming linear PCM signals, the conversion of signaling pulses to an out-of-band signaling format, and the generation of FDM pilots. An alternative approach to the implementation of these supplements is also given in Fig. 4. Finally, the TDM/FDM translator block TMUX containing directional filters is shown in detail in Fig. 5.

A. Multiple Single-Sideband Modulator

When converting continuous speech signals to PCM format, these signals must first be bandlimited by antialiasing filters. According to CCITT Recommendation G.712, the stopband constraints of these filters must be such that the effect of in-band spectral foldover is at a sufficiently low level. However, no provisions are made for out-of-band signaling centered at 3825 or 3850 Hz, respectively, which is specific to certain FDM systems. For this reason, the linear PCM signals must be subjected to an additional spectral shaping before the speech signals and the signaling waves are added at the input of the TDM/FDM translator TMUX. Moreover, in a 60-channel transmultiplexer, five (primary) group pilots and one supergroup pilot have to be introduced [17].

As a result of these considerations, a possible implementation of the complete multiple single-sideband modulator SSB-MOD is that shown in Fig. 3. First, all 60 linear PCM signals are processed by 60 identical bandpass filters $B_P^1$ operating at a sampling rate of 8 kHz. These bandpass filters have to meet the following (qualitative) requirements.

- Rejection of voiceband spectra in the signaling channels at frequencies about 3825 Hz/3850 Hz [16].
- Rejection of voiceband spectra of frequencies about 175 Hz/150 Hz. This is necessary since, after modulation, the equivalent frequencies of $-175$ Hz/150 Hz coincide with the adjacent signaling channels, thus causing noise [16].
- Rejection of voiceband spectra at the pilot frequency of 3920 Hz [16].
- Rejection of voiceband spectra at the frequency of 80 Hz. This is needed since, after modulation, the equivalent frequency of $-80$ Hz coincides with the pilot frequency of the adjacent channel, thus introducing noise [16].
- Complete rejection (attenuation pole) at the frequencies $f = 0$ and $f = 4$ kHz for suppression of (dc) components caused by residual carriers inherent in the FDM signal [18].

Immediately after the bandpass filter $B_P^1$, the out-of-band signaling waves and pilot frequencies are added to each channel. Out-of-band signals are generated by keying the amplitude (ASK = amplitude shift keying) of a carrier of 3825 or 3850 Hz, respectively, in correspondence to the incoming signaling pulses. In order to avoid distortion of the associated voiceband signals and pilot channels, the out-of-band ASK signaling waves must be subjected to a band limitation before they are added to the speech signals and the pilots. This is achieved by 60 bandpass filters $B_P^2$, which are all assumed to be identical (for specifications, see [16]). As indicated in Fig. 3, these bandpass filters may, for instance, be realized as interpolation filters [30], increasing step by step the sampling rates of the carriers of frequency 175 or 150 Hz, respectively. At the end of this interpolation process, the signals still have to be spectrally inverted (not shown in Fig. 3) to transpose the out-of-band signaling waves to the desired frequency range.

Next, the signals comprising speech, signaling waves, and pilots are fed into the 60 input ports of the TDM/FDM translator block TMUX. Here, in a multistep process, the FDM supergroup signal is derived from the baseband signals using a fundamentally single-way modulation scheme. In each stage, $l = 3$ or $l = 2$ signals, respectively, are combined, thus forming intermediate FDM signals containing a growing number of channels from stage to stage. At the same time, the sampling rate is increased by the corresponding factor of $l$; the intermediate rates are given in Fig. 3. Thus, the input sampling frequency of 8 kHz is increased by a total factor of $72 = 3^2 \cdot 2^3$ to $f_S = 576$ kHz at the output port of the TDM/FDM translator. Hence, the desired continuous FDM supergroup signal in the frequency range of 312-552 kHz can be directly derived from the TMUX output sequence, simply by using a D/A converter followed by an analog bandpass filter $B_P^A$. Because of the relation

$$312 \text{ kHz} - f_S/2 = f_S - 552 \text{ kHz} = 24 \text{ kHz},$$

corresponding to six unused channels on either side of the supergroup spectrum, sufficiently wide transition bands are available for the smoothing filter $B_P^A$ [6], [18].

As already mentioned, the block diagram of the multiple single-sideband demodulator SSB-DEMOD can be derived directly from Fig. 3 by transposition [2]. However, it must be noted that, even if the pilot frequency generator of the TDM/FDM translator is replaced by the associated transposed configuration of a frequency detector, the actual implementation of this detector cannot be deduced by transposition from the realization of the frequency generator. The same applies to the detection of out-of-band signaling waves. A thorough discussion of these questions will be given in Section III.

An alternative approach to the realization of the subassemblies for spectral shaping of the incoming linear PCM signals, the ASK conversion of the signaling pulses, and the generation (and detection) of the pilot frequencies is shown in Fig. 4. Here, every filter pair $B_P^1$ and $B_P^2$ of Fig. 3 is replaced by a directional filter containing complementary bandpass/bandstop filters. Again, these filters operate at a sampling rate of 8 kHz. Since the distortions of the pilot channels due to the ASK signaling waves can no longer be controlled by these
Fig. 3. Block diagram of the multiple single-sideband modulator SSB-MOD including signaling and pilot generation.

Fig. 4. Alternative approach to the implementation of the multiple single-sideband modulator SSB-MOD.
directional filters, additional bandstop filters $BS_p$ must, in general, be provided in the signaling paths of the six channels containing pilots. Furthermore, the carrier wave for ASK conversion of the signaling pulses must be generated at a sampling rate of 8 kHz. In contrast to Fig. 3, the pilots are introduced directly into the TDM/FDM translator at a suitable hierarchical level.

**B. TDM/FDM Translator**

An implementation of the TDM/FDM translator TMUX according to Fig. 4 is shown in Fig. 5. In this block diagram, the previously described multistage single-way modulation scheme of the TDM/FDM translator is clearly recognized. The blocks for frequency multiplexing of $l = 3$ or $l = 2$ intermediate (baseband) signals lend themselves to the application of directional filters which, at the same time, are used for increasing the sampling rate. Moreover, the (primary) group (PG) pilots and the supergroup (SG) pilots are introduced at different hierarchical levels following the first, second, and fifth stage, respectively, according to Fig. 4. Note that the frequency assignment to the various channels at the output ports of the directional filter blocks as well as the input and output sampling rates of each block are shown in Fig. 5. Finally, it should be mentioned that, in addition to Fig. 5, other sequential orders for the successive combination of $l = 3$ or $l = 2$ intermediate signals are conceivable.

The basic performance of the first (and correspondingly of the second) hierarchical stage for the combination of $l = 3$
baseband signals can be understood from Fig. 6. First, the sampling rate \( f_S = 8 \text{ kHz} \) of the incoming linear PCM signals is increased by a factor of \( l = 3 \) by inserting two zero-valued samples between every two adjacent samples of the original sequence. Thus, the spectra of the incoming signals are not changed [19]. Next, these spectra are subjected to individual filtering processes related to the sampling frequency \( f_{S3} = 3f_S = 24 \text{ kHz} \). In Fig. 6, \( H_{L3}, H_{B3}, \) and \( H_{H3} \) represent low-pass, bandpass, or high-pass filter characteristics, respectively. As a result, the spectrum \( S_3 \) of the intermediate FDM signal at the output of a first-stage directional filter is given by (Fig. 6)

\[
S_3 = S_{L3} + S_{B3}^I + S_{H3}
\]

(1a)

where

\[
S_{L3} \neq 0 \quad \text{for } 0 \leq f \leq 3f_S/6 = 4 \text{ kHz}
\]

\[
S_{B3}^I \neq 0 \quad \text{for } 3f_S/6 = 4 \text{ kHz} < f \leq 3f_S/3 = 8 \text{ kHz}
\]

\[
S_{H3} \neq 0 \quad \text{for } 3f_S/3 = 8 \text{ kHz} < f \leq 3f_S/2 = 12 \text{ kHz}
\]

Here, \( S_{B3}^I \) denotes the spectrally inverted replica of \( S_{B3} \). Obviously, in each bandpass branch \( B3 \), a spectral inversion is required in order to obtain the normal spectral position of all three signals involved in the resulting FDM spectrum \( S_3 \).

When processing the signals in the second stage, the input (baseband) signals are given by the intermediate FDM signals at the output port of the first stage. Hence, in (1), the sampling frequency \( f_S = 24 \text{ kHz} \) must be used at the input. Note that both in the upper and in the lower part of this stage, a complete directional filter block is replaced by a simple high-pass filter \( H3 \) for channels 1-3 and a simple low-pass filter \( L3 \) for channels 58-60, respectively. This is possible since only 60 instead of 72 PCM signals are actually translated to FDM format.

When comparing the filter specifications of the first and second stages, it will be recognized that the filter magnitude constraints of the second stage are more stringent than those of the first stage. In order to keep the number of different filter types as low as possible, aiming at a high degree of modularity, identical filters should be applied in both stages. This is achieved if, in both stages, filters designed for the second stage are used.

As shown in Fig. 5, in the third-fifth stages, \( l = 2 \) intermediate FDM signals are multiplexed, again by using directional filters. The associated spectral relations are obvious from Fig. 7. Since the spectra of both the low-pass (L) and the high-pass (H) filtered signals are required to appear in normal spectral position, the input signal of the high-pass branch (H) must be spectrally inverted before its sampling rate is increased. The output spectrum \( S_2 \) of a directional filter block for frequency multiplexing of \( l = 2 \) signals is given by a relation similar to (1):

\[
S_2 = S_L + S_H^I
\]

(2a)

where

\[
S_L \neq 0 \quad \text{for } 0 \leq f \leq 2f_S/4
\]

\[
S_H^I \neq 0 \quad \text{for } 2f_S/4 < f < 2f_S/2.
\]

(2b)

According to Fig. 7, \( f_S \) is again related to the input of the filter block being considered. In this case, too, identical filters can be utilized in all three stages if the filter design is based on the magnitude constraints of the fifth (last) hierarchical stage.

Thus, at the output port of the last stage, the complete 60-channel FDM signal is available in normal spectral position in the range \( 0 \leq f \leq f_{S\text{out}}/2 = 288 \text{ kHz} \). The frequency assignment of the channels is as follows. Channel 1: 24-28 kHz, \( \ldots \), channel 60: 260-264 kHz (normal position) or channel 60: 312-316 kHz, \( \ldots \), channel 1: 548-552 kHz (inverted position), respectively. In order to comply with CCITT Recommendation G.793 [17] requiring normal spectral position and natural sequential order of the frequency-to-channel assignment in the FDM supergroup spectrum, the output sequence of the last directional filter block is still multiplied by the sequence \((-1)^n(n \in \mathbb{N} \text{ related to the output sampling frequency } f_{S\text{out}} = 576 \text{ kHz})\) for spectral inversion, as shown in Fig. 5. In this way, the desired (post-DAC) FDM supergroup signal is obtained simply by using a bandpass filter \( BP_{60} \) for smoothing in compliance with Fig. 3. Channel 1: 312-316 kHz, \( \ldots \), channel 60: 548-552 kHz (normal position).

In contrast to the idealized filter characteristics of Figs. 6 and 7, in practice, only a finite minimum stopband attenua-
tion can be attained. As a consequence, crosstalk may occur between some channels. From Fig. 6, it can be seen that, for instance, the low-pass (L3) and the high-pass (H3) filtered signal spectra overlap at some frequencies with the same spectral orientation. Hence, the crosstalk between these channels is intelligible. On the other hand, crosstalk between a bandpass channel B3 and the associated L3 and H3 channels is unintelligible, since the spectrum processed in a bandpass branch is always spectrally inverted with respect to the signals in the two other branches. The same applies to the frequency multiplexing of \( l = 2 \) signals, as can be seen from Fig. 7. As a result, in any stage, \( l - 1 \) signals give rise to a crosstalk contribution to the channel under consideration. Since the TDM/FDM translator of Fig. 5 contains two stages with \( l = 3 \) and three stages with \( l = 2 \), one channel can be distorted by at most seven voiceband signals originating in contiguous channels. Of these, however, at most two signals contribute intelligible crosstalk to the channel under consideration. In addition, it should be noted that intelligible crosstalk can occur only if the signal being considered has passed at least one low-pass (L3) or/and one high-pass (H3) filter branch.

In Section II-A, two approaches to the introduction of the pilots into the FDM supergroup spectrum were discussed (cf. Figs. 3 and 4). From Figs. 5-7, it is obvious that the direct addition of the pilot carriers to the TMUX input signals would require expanded passbands for at least some of the TDM/FDM translator filters (e.g., the low-pass branch L3 of channel 25 for the SG pilot) if the pilots were to pass the TDM/FDM translator unaffected. If one succeeds, however, in introducing the pilot tones such that they are never located at a band edge of any subsequently processed intermediate FDM spectrum, then the presence of the pilots does not affect the TMUX filter specifications. This approach, corresponding to Fig. 4, is shown in Fig. 5. For example, the SG pilot is introduced into channel 25 following the first stage, where the added carrier frequency of \( f = 3.92 \) kHz is sampled at a rate of \( f_s = 24 \) kHz. Thus, it is far removed from the band edges of the subsequently processed intermediate spectrum, ranging from 0 to 12 kHz. Moreover, as shown in Fig. 5, four PG pilots can be introduced following the second stage at a sampling rate of \( f_s = 72 \) kHz, whereas the pilot of the third (primary) group can only be added after the last stage at the TMUX output operation rate of \( f_s^{\text{out}} = 576 \) kHz.

### III. EFFICIENT IMPLEMENTATION OF THE MODULES

As extensively discussed in the Introduction, only transversal (FIR) filters or wave digital filters (WDF [20]) are allowed for voiceband signal processing, since absolute stability under looped conditions is to be guaranteed. For this reason, the subsequent investigations on the implementation of the different modules and subassemblies of the multiple single-sideband modulator and demodulator are carried out by considering exclusively WDF or FIR filters.

#### A. Frequency Multiplexing of Two Baseband Signals

The individual operations for frequency multiplexing of \( l = 2 \) baseband signals according to Fig. 7 are schematically shown in Fig. 8. As can be seen from Fig. 7, the idealized transfer functions of the two branch filters are complementary with respect to each other according to the condition

\[
|H_L(e^{j2\pi f_s T/2})|^2 + |H_H(e^{j2\pi f_s T/2})|^2 = 1. \tag{3}
\]

First, let us consider the application of WDF's to our problem of frequency multiplexing as stated above. To this end, in Fig. 9 the scattering matrix of a WDF is recalled, showing the associated incident \( \{a_l\}_{l=1,2} \) and reflected \( \{b_l\}_{l=1,2} \) waves [20]. Here, \( S_{11} \) and \( S_{22} \) are the transmittances in either direction; \( S_{12} \) and \( S_{21} \) are the reflectances at the two ports of the filter. At real frequencies, the transmittances and reflectances of a reciprocal and pseudolossless WDF are related by Feldtkeller's equation [21], [22]:

\[
|S_{11}|^2 + |S_{21}|^2 = 1 \tag{4}
\]

where

\[
|S_{21}|^2 = |S_{12}|^2 \tag{5a}
\]

and

\[
|S_{11}|^2 = |S_{22}|^2. \tag{5b}
\]
Introducing, for instance, (5b) into (4), we get

$$|S_{21}|^2 + |S_{22}|^2 = 1.$$  (6)

Comparing (3) and (6), it is obvious from Figs. 7-9 that the frequency-multiplexed signal $s_2(kT/2)$ is obtained by means of a single-wave digital directional filter. If we assume

$$H_L(z) = S_{21}(z)$$  (7a)

and

$$H_H(z) = S_{22}(z),$$  (7b)

the low-pass and high-pass branch signals must be assigned to port 1 or port 2, respectively, according to the relations

$$a_1 = s_L(kT/2)$$

and

$$a_2 = s_H(kT/2).$$

Then the multiplexed signal $b_2 = s_2(kT/2)$ is obtained at port 2 of the WDF. Due to the high stopband constraints [16], however, the passbands must substantially be overspecified to comply with (6).

It should be noticed that, in our problem, according to Figs. 7 and 8, a particular situation arises: the magnitude constraints of the complementary transfer functions $H_L(z)$ and $H_H(z)$ are mutual mirror images with respect to a quarter of the sampling rate $f_{S2}$. As a result, the reflectance of the WDF is obtained from its transmittance just by inverting the frequency scale. It has been shown by Wegener [22] that for a symmetric wave digital directional filter, more than half of the adders and multipliers can be saved if it is implemented as a wave digital lattice filter, provided that the canonic lattice impedances are realized by cascaded all-pass sections or by cascaded unit elements.

Even though a wave digital directional filter is expected to be the best choice for the implementation of the filter block of Fig. 8, let us briefly take a closer look at an appropriate transversal realization of the transfer functions $H_L(z)$ and $H_H(z)$ which preserves the single-way modulation scheme. Note that these transfer functions no longer need be strictly complementary according to relation (3). The high-pass filter $H_H(z)$ in the lower branch of Fig. 8 is replaced by a low-pass filter followed by a multiplier for spectral inversion. Here, the transfer function of this low-pass filter is identical to $H_L(z)$ of the upper branch (Fig. 10). The low-pass transfer function $H_L(z)$ should preferably be minimum phase.

Fig. 11 shows an efficient implementation of a minimum phase FIR filter for increasing the sampling rate by an integer factor $m(f_S = 1/T)$.

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**Fig. 9.**

**Fig. 10.** Alternative approach to Figs. 7 and 8 ($T = f_{S2}/2$).

**Fig. 11.** Efficient implementation of minimum phase FIR filter for increasing the sampling rate by an integer factor $m(f_S = 1/T)$. 
WDF's seem to be more suitable. This assumption is based on the fact that the prescribed filter constraints are very stringent [16].

B. Frequency Multiplexing of Three Baseband Signals

On the basis of the preceding considerations concerning the application of wave digital directional filters for frequency multiplexing of two signals, frequency multiplexing of $l = 3$ baseband signals can be performed very efficiently (Fig. 12). First, the low-pass and high-pass branch signals $s_{L3}(kT/3)$ and $s_{H3}(kT/3)$ are premultiplexed after the associated input sampling rate has been increased by a factor of $l = 3$ (filter $L3H3$). Again, a wave digital directional filter can be used as a multiplexer. Here, $|H_1|$ denotes the complementary magnitude response related to $|H_1|$ [Fig. 12(b)]. Afterwards, the resulting intermediate signal $s_{I3}(kT/3)$ is combined with the spectrally inverted bandpass branch signal $s_{B3}(kT/3)$, which is related to the output sampling rate $f_{S3} = 3f_S = 3/T$. Here, according to Fig. 12, multiplexing is performed by use of a bandpass-type magnitude response $|H_2|$ for processing of $s_{B3}(kT/3)$. The premultiplexed signal $s_{I3}(kT/3)$ is spectrally shaped by the associated complementary bandstop characteristic $|H_2|$. Comparing Figs. 6 and 12(b), the individual branch transfer functions of the first two TMUX stages and those of the wave digital directional filters of Fig. 12 are related by

$$H_{L3} = H_1 \cdot H_2$$

(8a)

$$H_{B3} = H_2$$

(8b)

$$H_{H3} = H_1 \cdot H_2$$

(8c)

Note that the transfer functions $H_1$ and $H_1$ are again mutual mirror images with respect to $f_S / 4$. In addition, the transition bands

$$\Delta f \gg f_{S3} / 6 = f_S / 2$$

of these filter transfer functions are very wide [Fig. 12(b)], thus requiring only low filter orders. This is true despite the fact that their stopband constraints have to comply with the more stringent requirements for suppression of intelligible crosstalk (cf. Section II-B).

In constrast to the filter block $L3H3$, both magnitude responses $|H_2|$ and $|H_2|$ are symmetrical about $f_S / 4$. Therefore, the bandpass transfer function $H_2(z)$ can be derived from a prototype low-pass filter function $G(z)$ simply by replacing the argument of the latter by $-z^2$ [24] according to the relation

$$H_2(z) = G(-z^2).$$

(9)

This operation corresponds to a warping of the frequency axis according to

$$\frac{f_G}{f_S} = \frac{f_H}{f_S} = \frac{1}{2}.$$  

(10)

Here, $f_G$ denotes the actual sampling rate; $f_G$ is related to the low-pass transfer function $G(z)$ and $f_H$ to the bandpass characteristic $H_2(z)$ according to (9). Applying (10), the passband frequency range

$$\frac{1}{6} < \frac{f_G}{f_{S3}} < \frac{1}{3}$$

of $H_2(z)$ is transposed to the frequency range

$$\left| \frac{f_G}{f_{S3}} \right| < \frac{1}{3}$$

of the prototype filter $G(z)$. As a consequence of (9), the computational complexity of the bandpass filter $H_2(z)$ is the same as that of the associated low-pass filter $G(z)$; only the number of delays is doubled. The above considerations are applicable
to WDF's and FIR filters [24]. However, a transversal implementation of the transfer functions $H_2$ and $\bar{H}_2$ is substantially less efficient than a wave digital directional filter approach since the transition bands of the associated magnitude responses are narrow, requiring a high filter order. Furthermore, the sampling rate alteration capability of the minimum phase FIR filters of Fig. 11 cannot be exploited for reducing the multiplication rate.

Finally, an implementation of the filter block for frequency multiplexing of $I = 3$ baseband signals exclusively using FIR filters is shown in Fig. 13. In each branch, the filter magnitude responses of Fig. 6 are directly realized. The low-pass and high-pass branch signals are processed according to Figs. 10(a) and 11.

C. Spectral Shaping at the Front End of the TDM/FDM Translator

In Section II, it was conjectured that the spectral shaping of the voiceband signals and signaling waves at the input of the TDM/FDM translator could advantageously be performed by applying directional filters (Fig. 4). For this purpose, we propose WDF's, although the passband magnitude response of the voiceband filter branch must be substantially overspecified, and the complementary transfer functions are no longer mutual mirror images with respect to one quarter of the sampling frequency. However, the voiceband filter magnitude constraints may be rendered symmetrical with respect to a quarter of the sampling rate, as suggested by Pelloni [18]. Then the bandpass transfer function can be derived from a low-pass prototype function again by applying the spectral transformation of (9) if the stopbands and the upper transition band of the filter are somewhat overspecified.

D. Signaling

The following discussion on signaling is based on Fig. 4. Here, the generation of FDM out-of-band signaling is accomplished by modulating the amplitude (ASK) of a carrier of frequency $f_c = 3825$ Hz or $f_c' = 3850$ Hz, respectively, corresponding to the signaling pulses. A very efficient realization of the carrier generation is obtained by storing the appropriately quantized samples of a 3825 Hz (3850 Hz) sinusoidal wave in a read-only memory (ROM), requiring just a simple $N$ counter with increment 1 for address calculation. Its clock rate is fixed to $f_s = 8$ kHz. Since the highest common subharmonic frequency of the carrier frequency $f_c(f_c')$ and the associated sampling frequency $f_s$ is given by $f_0 = 25$ Hz ($f_0' = 50$ Hz), a total of $N = 320$ ($N' = 160$) samples must be stored in the memory. These figures may, for example, be reduced to 160 (80) by introducing an additional sign control unit, as can be seen from the relation

$$\sin \left(\frac{2 \pi k 3825}{8000}\right) = \sin \left(\frac{2 \pi k 153}{320}\right)$$

$$= \sin \left[2 \pi (160 + i) \frac{153}{320}\right]$$

$$= -\sin \left(2 \pi i \frac{153}{320}\right).$$

A somewhat different realization scheme is obtained if the samples of the highest common subharmonic frequency $f_0 = 25$ Hz ($f_0' = 50$ Hz) are stored instead of $f_c(f_c')$ [26].

When transposing the block diagram of Fig. 4 for extracting the signaling pulses from the FDM signal, the modulator switches must be replaced by frequency detectors. A simple realization of such a subunit is shown in Fig. 14. Here, the bandpass filter block $BP$ is dashed to indicate that this filtering operation is already accomplished by the wave digital directional filter for voiceband and signaling channel separation (and by the pilot bandstop filter). The following digital rectifier merely suppresses the sign of the incoming signal. Finally, the rectifier output sequence is smoothed by a decimating low-pass filter according to Fig. 15 [26]. Thus, the operating rate is reduced to 500 Hz by a factor of 16. A threshold device regenerates the baseband signaling pulses to be introduced into the common PCM signaling channel.

E. Generation and Detection of the Pilots

The generation and detection of the pilots of Figs. 3 or 4, respectively, can be accomplished in the same or a similar way as was described for signaling in the preceding section. Here, we only want to deal with the multifrequency approach of Fig. 4.

First, the most obvious solution is to use a separate ROM for the generation of each pilot tone indicated in Fig. 5. The associated amount of storage is given in Table I. Here, no use is made of any symmetry. A first reduction by half the storage locations is achieved by introducing a sign inverter on the basis of (11). Next, as can be seen from Table I, only three different operating rates occur. Hence, it is sufficient to mechanize only three ROM's for pilot tone generation, one for each sampling frequency $f_s$.

In the case of $f_s = 72$ kHz, the quantized samples of the highest common subharmonic wave $f_0 = 80$ Hz must be stored [26]. This frequency is related to $f_s$ by

$$f_0 = f_s/N = f_s/900$$

where the intermediate pilot frequency being introduced is
is easily achieved by successively reading every \((F \cdot k)\)th sample from a 80 Hz sine table related to \(f_S\) where \(k = 0, 1, 2, \ldots\). According to (14), the calculation of the argument \((F \cdot k)\) must be carried out \(\mod N\). The frequency factors \(F\), related to the common basic frequency \(f_0 = 80\) Hz, are included in Table I.

Finally, the number of different sine tables (ROM's) can be reduced to only one if the desired pilot tones are derived by an interpolation or a decimation process from a common memory. However, the implementation of the interpolation approach, needing only a small memory, generally requires a substantial amount of circuitry for filtering and address calculation. Therefore, the application of a decimation algorithm is more efficient. To this end, a quarter period of a sine wave of the basic frequency \(f_0 = 80\) Hz, sampled at the highest operating rate of \(f_S = 576\) kHz, may be stored in a memory, thus necessitating a total of \(N/4 = 1800\) storage locations. Using (12)-(14), the samples of a sine wave of frequency \(f = F \cdot f_0\), which is related to the generally lower sampling rate

\[
f_S' = f_S/L \quad L \in \mathbb{N},
\]

can be read from the above 80 Hz sine table (related to \(f_S\)) by taking every \((F \cdot L \cdot k)\)th entry \(\mod N\):

\[
\sin \left(\frac{2\pi k F \cdot f_0}{f_S'}\right) = \sin \left(\frac{2\pi F \cdot L \cdot k}{N}\right)
\]

where \(k = 0, 1, 2, \ldots\). (Note that the actual set of addresses must be reduced to 1, 2, ..., \(N/4\)). Here, no filtering is necessary. The amount of circuitry for address calculation necessitating different accumulators, an up-and-down counter, and a sign inverter is moderate.

When transposing the block diagrams of Figs. 4 or 5, respectively, the detection of the pilots can be carried out according to Fig. 14, where the actual operating rates of Fig. 5 have to be inserted. Note that, for this application, the dashed bandpass filter block \(BP\) (Fig. 14) for rejection of speech and signaling spectra of adjacent channels must be explicitly implemented. Since some of these frequency detectors have to operate at high rates, the associated bandpass filters are subjected to stringent constraints. These requirements may, however, be alleviated by rigorously exploiting all inherent sample rate reduction capabilities [27]-[29].

### IV. FILTER DESIGN

As stated in the previous section, an implementation of the TMUX filters (Fig. 5) as directional WDF's is more efficient than a minimum phase FIR filter realization. Therefore, only the design results of the three different WDF's needed throughout the TDM/FDM translator are presented here.

The structures of all three WDF's are of the lattice type, whose canonic impedances are realized with cascaded unit elements [22], [31]. Therefore, we are able to benefit from the reduced number of multipliers and adders required, which follows from the symmetric magnitude constraints set up in Section III. In contrast to most other WDF implementations, these structures allow a high degree of parallel processing and, thus, lend themselves to high-speed applications [31].

Fig. 16(a) shows the directional WDF \(L3H3\) of degree 9 and Fig. 17(a) the directional bandpass WDF \(B3\) of degree 38 according to Fig. 12. (For the meaning of adaptor symbols, the reader is referred to [20].) In Fig. 16(b) and Fig. 17(b), the magnitude responses in both directional filter branches are shown. The solid lines show the magnitude responses with exact coefficients and the dashed lines the magnitude responses with quantized coefficients (Fig. 16(b): 12 bits, Fig. 17(b): 16 bits). Finally, Fig. 18 shows the directional WDF \(L2H2\) of degree 25 used to multiplex two baseband signals according to
Fig. 16. Lattice directional WDF L3H3 of degree 9, $f_0 = 72$ kHz. (a) Wave flow diagram. (b) Attenuation in the low- and high-pass branch; solid line: nonquantized coefficients, dashed line: coefficient word length 12 bits.

On the basis of the above filter design, we are able to present two important features of our digital transmultiplexer approach. The first one is concerned with the computational load of the TDM/FDM translator. In Table II, the complexity of each filter and of each hierarchical stage is summarized in detail, distinguishing between multiplication and addition rate. The total computational load on a per-channel basis amounts to $42 \cdot 10^6/60 = 0.7 \cdot 10^6$ multiplications/(second-channel) and $116 \cdot 872 \cdot 10^6/60 \approx 1.95 \cdot 10^5$ additions/(second-channel).

The second feature is the low absolute value of the total group delay of the transmultiplexer. In Fig. 19, the group delay of the channel with the highest absolute value of the group delay is depicted with the associated tolerance scheme taken from CCITT Recommendation G.792 [16]. As can be seen, this tolerance scheme is met with a great margin. The minimum group delay measured at the analog ports of the transmultiplexer, the digital ports being looped, becomes $2\tau_{\text{min}} = 1.28 \text{ ms}$. This figure favorably compares with the maximally tolerated value of 3 ms [16].

V. CONCLUSIONS

In this paper, a multistage approach to the implementation of a digital transmultiplexer was proposed. Its TDM/FDM translator, which was described in detail, is based on a single-way modulation scheme. It contains directional filters for frequency multiplexing of an increasing number of signals from stage to stage. Hence, in contrast to most of the transmultiplexer approaches published so far, our transmultiplexer needs neither processors for orthogonal transforms (such as DFT, FFT, or Hadamard transforms) nor modulators (except those used for the trivial operation of spectral inversion).

The computational load of the proposed TDM/FDM translator requiring a multiplication rate on a per-channel basis of 700 000/s compares well with that of the Tsuda approach [14], which has to perform 760 000 multiplications per second and channel. This holds true despite the higher sampling rate of 576 kHz (Tsuda: 512 kHz) used at the FDM end. Thus, in contrast to [14], a supplementary analog frequency conversion of the composed FDM signal is avoided. Subsequently, three additional merits of the novel single-way modulation scheme are particularly emphasized. First, our approach exhibits a high degree of modularity, meaning that the transmultiplexer is composed of only a small number of different modules. Actually, the TDM/FDM translator contains a certain number of only three distinct filter types; the same three types are used exclusively in the transposed structure for FDM to TDM translation. Due to its high degree of modularity and the complete absence of processors for orthogonal transforms and modulators, the amount of circuitry necessary for control units and auxiliary (shimming) delays (overhead cir-
Fig. 17. Bandpass lattice directional WDF $B_3$ of degree 38, $f_s = 72$ kHz. (a) Wave flow diagram. (b) Attenuation in the bandpass and bandstop branch; solid line: nonquantized coefficients, dashed line: coefficient word length 16 bits.

...is expected to be very small. For all these reasons, the proposed transmultiplexer approach lends itself to the application of LSI/VLSI implementation techniques. As a further consequence of its high degree of modularity, manufacturing and test procedures are greatly facilitated [5], [6]. Second, in order to guarantee absolute stability, even under looped conditions with only a small amount of additional hardware [5], [7], the proposed transmultiplexer, being based on a single-way modulation scheme, is exclusively composed of wave digital filters (WDF). Moreover, these filters are very efficient due to the symmetry properties of their magnitude responses. Third, only minimum phase WDF's are used. As a result, the lowest absolute value of the group delay measured at the analog ports of the TDM/FDM translator (the digital ports being looped) which has been reported on TMUX is obtained: 1.28 ms, which is far below the allowed value of 3 ms [16]. This is additionally supported by the absence of transform processors and modulators. The impact of supplements, e.g., for signaling, pilot generation and detection, and spectral shaping of the incoming linear PCM signals, is treated in detail.

Finally, let us briefly discuss two modifications of the transmultiplexer approach proposed in this paper, which use a sampling rate of 512 kHz at the FDM end, thus requiring an additional analog frequency translation [32]. The first one is composed exclusively of the $L2H2$ directional filter of degree 25 as described in Section IV. As a result, the tree-like multi-stage structure of this TDM/FDM translator, which is similar to Fig. 5, exhibits the highest attainable degree of modularity with all associated merits [5], [6]. Furthermore, the multi-
Fig. 18. Lattice directional WDF $L2H2$ of degree 25, $f_S = 576$ kHz.
(a) Wave flow diagram. (b) Attenuation in the low- and high-pass branch; solid line: nonquantized coefficients, dashed line: coefficient word length 16 bits.

### TABLE II
**REQUIRED FILTERS AND COMPUTATIONAL LOAD OF THE TDM/FDM TRANSLATOR**

<table>
<thead>
<tr>
<th>Theoretical stage</th>
<th>Filter</th>
<th>Filter degree</th>
<th>Number of multipliers</th>
<th>Number of odders</th>
<th>Sampling frequency [kHz]</th>
<th>Number of filters required</th>
<th>Operation rate</th>
<th>multiplications per channel</th>
<th>Additions per channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>L2H2</td>
<td>9</td>
<td>4</td>
<td>15</td>
<td>26</td>
<td>20</td>
<td>$1.92 \times 10^7$</td>
<td>$7.2 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>15</td>
<td>60</td>
<td>26</td>
<td>10</td>
<td>$5.12 \times 10^7$</td>
<td>$29.8 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>L2H2</td>
<td>25</td>
<td>12</td>
<td>35</td>
<td>164</td>
<td>6</td>
<td>$8.208 \times 10^7$</td>
<td>$25.92 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>12</td>
<td>39</td>
<td>288</td>
<td>2</td>
<td>$5.915 \times 10^7$</td>
<td>$22.481 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>L2H2</td>
<td>25</td>
<td>12</td>
<td>39</td>
<td>575</td>
<td>1</td>
<td>$5.912 \times 10^7$</td>
<td>$22.481 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L2H2</td>
<td>25</td>
<td>12</td>
<td>39</td>
<td>575</td>
<td>1</td>
<td>$5.912 \times 10^7$</td>
<td>$22.481 \times 10^5$</td>
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<tr>
<td></td>
<td></td>
<td>25</td>
<td>12</td>
<td>39</td>
<td>575</td>
<td>1</td>
<td>$5.912 \times 10^7$</td>
<td>$22.481 \times 10^5$</td>
<td></td>
</tr>
</tbody>
</table>

Operation rate on a per channel basis:

- multiplications per channel: $4.7 \times 10^7$
- additions per channel: $1.95 \times 10^7$
multiplication rate per channel is reduced to 600 000/s. Hence, Tsuda’s method—now directly comparable—is by far outperformed by this approach in all aspects discussed here. If, on the other hand, the multiplication rate is required to be minimum, then the directional WDP’s must be designed individually for each of the six successive stages, avoiding any overspecification. In this case, the filter order increases from 13 at the PCM end to 25 at the FDM end, where the latter filter is identical to the \(L2H2\) filter (Fig. 18). Nevertheless, all directional WDP’s are of the same basic \(L2H2\) structure. The only difference between the six individual filters is due to the different orders of their canonical impedances, resulting in different lengths of the two-port adaptor chains in Fig. 18(a). An implementation of this transmultiplexer approach must perform only 466 000 multiplications per second and channel.

ACKNOWLEDGMENT

The authors are greatly indebted to B. Birn for many stimulating discussions concerning filter design and to H. Eyssel for the thorough analyses of the designed filters. Furthermore, they would like to express their gratitude to Dr. Gleissner, and particularly to Prof. Temes (University of California, Los Angeles), for their invaluable constructive criticism in reviewing the manuscript.

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