On the design of digital and time-discrete modem transmitters with linearly modulated data waveforms

H. Göckler*+**

In this paper, we focus on the design of time-discrete and digital modem transmitters (Echo modulation) with linearly modulated data waveforms. We formulate a minimization problem, which can be solved directly to the signal elements or to their equivalent baseband representation. To this end, let the symmetry properties of the signal space be exploited. Thus, the efficiency of this design method is shown by way of a digital pseudo 8-phase DPSK modem transmitter according to CCITT Recommendation V.26 bis.

1 Introduction

In recent years, the development of data transmission systems (modems) has proceeded steadily. An important step towards the realization of small and relatively cheap modems was the replacement of the inductance-capacitance (LC) filters by binary transversal filters (BTF), which, in general, were composed of flip-flops and resistors [1-3]. Recent developments of large scale integration (LSI) techniques have opened new technological possibilities, which have stimulated research into the all-digital implementation of data modems [4-11]. Digital structures are easier to implement as an LSI circuit than analog circuits. Furthermore, a digital modem can be programmed merely by changing the contents of memories [5-10].

When designing commercial data modem transmitters, the prescribed specifications are usually based on the CCITT Recommendations. Thus, for a modem transmitter to be developed for commercial applications, in most cases a fixed transmission rate is specified in addition to the modulation method. Moreover, reduced rate capabilities may be prescribed. For this reason, the transmitted modulation can specifically be adapted to the particular task, thus reducing both material requirements and time expenditure for alignment and testing during production. Therefore, the digital echo modulation — first proposed by Croisier and Pierret [4] — and associated approaches [8-10] lend themselves to the digital realization of a data transmitter. Following a paper by Kammerer and Schenk [12], the principle of operation of a modem transmitter will briefly be recalled in section 2 in order to set up a transmitter model, which can serve as a basis for the subsequent statement of the data transmitter design problem to be solved in this contribution. Finally, the design of a pseudo 8-phase DPSK modem transmitter for the transmission of 2400 bit/s over voiceband telephone channels according to the CCITT Recommendation V.26 bis is taken as an example.

2 The transmitter model

At first, Fig. 1 shows the model of a general time-discrete data transmitter for arbitrary linear modulation forms, which is familiar from analog designs. From the serial binary data stream, blocks of 7 bits are formed and fed into an encoder in parallel form, which provides a corresponding pair of numbers {a_k, b_k} at the desired symbol rate on the basis of a one-to-one relationship. The complete set \{a_\lambda, b_\lambda | \lambda = 1, 2, \ldots, A\}, where generally \lambda = 2^f, may be regarded as Cartesian representation of all signals in the baseband signal space. Thus, the modulation method is determined by the encoder. After sampling at the symbol rate 1/T and lowpass filtering by two identical pulse-shaping networks with the impulse response g(kT_b), the modulation is carried out by multiplication with two orthogonal carriers cos(\omega_k k T_b) and sin(\omega_k k T_b). Here, the sampling frequency f_s = 1/T requires a symbol rate of 1/T_b. Thus, the symbol rate must remain constant in order to maintain the sampling theorem. Moreover, it must be governed by the desired spectral characteristics of the transmitter output sequence. Subsequently it is, for convenience, always assumed that the sampling frequency is a multiple integral of the symbol rate. For vestigial sideband modulation (VSB), an additional filter with the impulse response g_{VSB}(k T_b) serves for suppression of the unwanted sideband. In this case, b_1 must be set to b_1 = 0.

Before continuing the discussion of the transmitter model, the conditions for ideal data transmission (as given in [13]) are briefly recalled for better understanding of the subsequent statement of the modem transmitter design problem. To this end, let g_{VSB}(k T_b) = \delta_0 (k T_b) or G_{VSB}(\exp (j2\pi f_0)) = 1, respectively, and assume the transmission path (including the modulation and demodulation process) to be free from linear and nonlinear distortions. Then, nonerroneous data transmission is theoretically possible, if the two identical pre-modulation pulse-shaping filters of Fig. 1 represent ideal lowpass filters with unity gain up to the cutoff frequency f_N = 1/2 T and zero transmission beyond f_N. In addition phase shift proportional to frequency is assumed. In this case, g(k T_b) is a \sin(\pi x)/x.

* Dipl.-Ing. Heinz Göckler, VDE/NTG, ANT Nachrichtentechnik GmbH, Backnang
** Manuscript received: March 5, 1984
type of impulse response of infinite duration with equidistantly spaced crossings of the time axis at times $\pm i T = \pm i/(2 f_N)$. Thus impulses can be transmitted at such intervals without interference between the peaks of the received pulses. The reciprocal of 2

\[ \frac{1}{2} \]

aliasing parameter $T_s$ deteriorates, with a more gradual modification retains the cutoff frequency of the lowpass characteristic, the oscillatory nature of the pulse tails is reduced. The original sharp cutoff is modified by an amplitude characteristic having odd symmetry about the cutoff frequency $f_N$. This modification retains the zero points of the $(\sin x)/x$ response and adds certain others.

In contrast to the above ideal conditions, a practically implemented modem transmitter suffers from various deteriorations, such as deviations of the pulse-shaping filters from the desired frequency response (linear distortions), aliasing effects due to nonideal out-of-band rejection of the lowpass filters and to nonlinear modulator performance, and quantization effects. While linear distortions are well understood from a frequency domain description, nonlinear distortions must be treated in the time domain. This should be kept in mind for the statement of the modem transmitter design problem to be presented in section 3.

Returning to the development of the transmitter model, various approaches deviating from that of Fig. 1 have proved to be more efficient for a digital implementation of a modem transmitter [5–10]. To this end, the transmitter output sequence $x(k T_S)$ of Fig. 1 is expressed in complex notation:

\[ x(k T_S) = \sum_{i=-\infty}^{\infty} \text{Re} \left\{ c_i \hat{g}(k T_S - i T) e^{-j \omega_c k T_S} \right\} \hat{g}_{\text{VSB}}(k T_S) \]  

(1)

where $c_i = a_i + j b_i$ and the asterisk (*) represents the convolution operator. Defining the complex signal

\[ u(k T_S) = u_1(k T_S) + j u_2(k T_S) \]

\[ = [\hat{g}(k T_S) e^{-j \omega_c k T_S}] \hat{g}_{\text{VSB}}(k T_S), \]  

(2)

after a simple rearrangement of eq. (1), there follows

\[ x(k T_S) = \sum_{i=-\infty}^{\infty} \text{Re} \left\{ c_i e^{-j \omega_c T_S} u(k T_S - i T) \right\}. \]  

(3)

This formulation is known as "Digital Echo-Modulation" or "Microred Modem Transmitter", respectively [4–6]. Essentially, it says that the complex data $c_i$ are rotated by the continuously (or mod $2 \pi$) increasing angle of

\[ \varphi_i = \omega_c T i = 2 \pi f_c T i = 2 \pi (i/(L)) i, \quad i, L \in \mathbb{N}. \]  

(4a)

Note that $\varphi_i$ is determined by the ratio of the carrier frequency $f_c$ to the symbol rate $1/L$, which is subsequently assumed to be a rational, relatively prime number $1/L$. Hence, $L$ represents the number of different rotation angles occurring. According to eq. (3) the complex signal $u(k T_S)$, which comprises all transmitter filter responses, is weighted by the rotated data symbols (Fig. 2a).

According to eq. (2), the two orthogonal components of the complex signal $u(k T_S)$ define the signal space. Therefore $\{u_1, u_2\}$, a set of two signal vectors $^1$, may be interpreted as orthogonal base. The symbol dependent (subscript $i$) development coefficients are obtained from eq. (3) by the real and imaginary part

\[ y_i = \alpha_i + j \beta_i = c_i e^{-j \omega_c T_S} = c_i e^{-j 2 \pi i/(L)} i. \]  

(5)

The total number $M$ of different values $\gamma_\mu$, $\mu = 1, 2, \ldots, M$, is limited by

\[ \Lambda \ll M \ll \Lambda L \]  

The upper bound is obvious from Fig. 1 and 2, and from eqs. (4) and (5). The lower bound is obtained if each rotated complex symbol $\gamma_\mu$, $\mu$, coincides with one of the unrotated baseband symbols $\{c_\lambda \mid \lambda = 1, 2, \ldots, A\}$.

If, as mentioned previously, both the carrier frequency and the symbol rate are fixed, the complete set of different complex values $\{\gamma_\mu \mid \mu = 1, 2, \ldots, M\}$ may be precomputed according to eq. (5) and stored, if $M$ is not too large an integer. If, in addition a finite duration of $u(k T_S)$ is assumed, the base vectors $u_1$ and $u_2$, weighted by $\alpha_i$ and $\beta_i$, respectively, can be stored and superimposed according to the symbols to be transmitted. Thus, no multiplications must be carried out in the modem transmitter. Finally, the most efficient arrangement is obtained, if the $M$ signals

\[ s_\mu(k T_S) = \text{Re} \left\{ c_\lambda e^{-j 2 \pi i/(L)} u(k T_S) \right\}, \]  

(6)

where

\[ \mu = 1, 2, \ldots, M \quad (i = 0, 1, \ldots, L - 1; \lambda = 1, 2, \ldots, A). \]  

\(^1\) Vectors are denoted by underlining.
are stored in a read-only memory (ROM) and superimposed as time-delayed replica of $s_\mu (k T_S)$ according to eq. (3), as it is shown in Fig. 2b. If the tails of $n$ signals according to eq. (6) overlap, the memory read-out and accumulation process must be performed at the rate $n f_S$.

Note that due to signal space symmetries the number $m$ of signal vectors $s_\mu$, which must necessarily be stored, is generally smaller than $M$ [14]:

$$m < M.$$  

The assignment of the signals $s_\mu, \mu = 1, 2, \ldots, m$ to the symbols $c_i$ to be transmitted in the $i$th slot considering the actual phase rotation by $\phi_i$ according to eq. (4) may be accomplished by a simple phase accumulator and address computer, as outlined in Fig. 2b. Clearly, the number of storage locations is of minor impact on the hardware costs due to the availability of low-cost ROMs. Generally, a trade-off must be made between storage savings and the increase of control complexity. Nevertheless, we are greatly interested in small figures of $m$, since only $m$ independent signal vectors $s_\mu$ have to be considered in the data transmitter design algorithm to be described in the following section.

3 Statement of the design problem

Subsequently, the signals $s_\mu$ according to eq. (6) will be referred to as elementary bandpass signals or simply as signal elements (SE). From eqs. (1) and (3) and from Fig. 1 and 2 it is obvious that all $m < M$ signal elements can be determined for arbitrary modem transmitter implementations, finite $M$ provided. Then, the subsequent superposition of $n$ SEs in order to compose the transmitter output sequence $x(k T_S)$ is carried out free of (additional) errors, if the signal elements are scaled such that any overflow is avoided (cf. Fig. 2b). The design of a digital or time-discrete modem transmitter of any structure can thus be reduced to a statement of the design problem common to all structures: The filters and other networks for time-discrete signal processing in the modem transmitter have to be optimized such that the resulting signal elements optimally meet certain time and frequency domain requirements still to be specified. For a digital implementation, quantization effects must be taken into account in the determination of the SEs.

Considering different aspects, Croisier and Pierret [4], Choquet and Naustbauer [5], Kammeyer [15] and Schenk [16] have dealt with the design of SEs or the associated orthogonal base $\{u_1, u_2\}$, respectively. However, finite word length and nonlinear effects have essentially been neglected so far.

Generalizing recent results [14], this paper aims at the description of a procedure for the design of time-discrete modem transmitters, whereby finite wordlength effects may be taken into account. This algorithm is based on the generalized problem stated above, for which reason it is suited for the design of both digital and time-discrete data transmitters of any structure and with any linear modulation form, provided $f_c T = 1/L, 1, L \in \mathbb{N}$.

According to eq. (7), generally only $m < M$ signal elements must be stored for the implementation of a modem transmitter as shown in Fig. 2b, or introduced in the data transmitter design algorithm, respectively. This holds true if it is possible to derive an arbitrary subset of SEs $E_1 \subseteq E$ free from errors from another subset $E_2 \subseteq E$ by making use of the symmetry properties of the signal space, provided that $E_1 \cap E_2 = \emptyset$. Here, $E$ represents the set of all $M$ SEs appearing in the signal space and $\emptyset$ the empty set.
First kind of symmetry: In Fig. 3a the arrangement of the SEs is symmetrical to the origin of the phase plane (signal space). Obviously, from the subset \( E_1 = \{ s_\mu \mid \mu = 1, 2, 3, 4 \} \) all other SEs can be derived according to the symmetry relation:

\[
\begin{align*}
\hat{s}_{\mu+4} &= -s_\mu, & \mu &= 1, 2, 3, 4.
\end{align*}
\]  

Second kind of symmetry: The rotated arrangement of SEs as shown in Fig. 3b is, in addition, symmetrical to both coordinate axes. In this case, for instance, the SE \( s_8 \), axis-symmetrical to \( s_2 \), can be derived from \( s_2 \) by rearranging the sequential order of the \( N \) components of the vector \( s_2 \) according to the transformation:

\[
\begin{align*}
\hat{s}_8 &= P \cdot s_2.
\end{align*}
\]  

Here, \( P \) is an \( N \times N \) permutation matrix of the form

\[
P = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]  

The rearrangement of the signal vector \( s_2 \) due to this matrix transformation corresponds to a time reversal of the sequence \( s_2, (k T), \) where \( k \) runs from 1 through \( N \). The proof that eq. (9) meets the desired symmetry requirements is given in Appendix A. With the signal arrangement of Fig. 3b, the subset \( E_2 = \{ s_\mu \mid \mu = 1, 2, 3 \} \) only comprises three SEs because of eq. (9). In particular, the following symmetry relations hold:

\[
\begin{align*}
\hat{s}_4 &= -P \cdot s_2, \\
\hat{s}_{\mu+4} &= -s_\mu, & \mu &= 1, 2, 3, \\
\hat{s}_8 &= P \cdot s_2.
\end{align*}
\]  

The most efficient arrangement of SEs in the phase plane for this 8-PSK system is depicted in Fig. 3c. Here, the subset \( E_3 \) comprises the minimum number of \( m = 2 \) SEs, for instance \( E_3 = \{ s_1, s_2 \} \). Thus, the SEs of the complementary subset \( E_2 \) can be derived as follows:

\[
\begin{align*}
\hat{s}_3 &= -s_7 = -P \cdot s_2, \\
\hat{s}_4 &= -s_8 = -P \cdot s_1, \\
\hat{s}_{\mu+4} &= -s_\mu, & \mu &= 1, 2.
\end{align*}
\]  

Hence, in this case, only \( m = 2 \) SEs have to be considered in the modem transmitter design procedure. Depending on the relative control complexity, in the signal element memory of Fig. 2b \( M = 8 \), or four, or \( m = 2 \) SEs may be stored.

For a more profound understanding it must be noted that, for instance, in the case of Fig. 3c not any two of all \( M \) SEs can be combined to form the signal set \( E_1 \). If we choose, for example, \( E_1 = \{ s_1, s_8 \} \), where \( s_8 \) can be derived from \( s_2 \) according to eq. (12), the SEs \( \{ s_\mu \mid \mu = 2, 3, 6, 7 \} \) cannot be obtained from \( E_1 \). For this reason, the SEs forming the signal subset \( E_1 \) must not be derivable from each other by one of the above symmetry relations.

At the beginning of this section a generalized statement of the design problem of digital modem transmitters was given. If the solution of the problem defined this way were to be pursued in accordance with the above formulation, depending on the chosen structure (cf. Fig. 2) different...
parameters of the transmitter, such as the orthogonal base \( \{u_1, u_2\} \) or the SEs, would have to be optimized. Considering, however, that all transmitter structures suitable for a digital implementation can be derived from the model depicted in Fig. 1 by use of eqs. (1)-(3) and eq. (6), then the design problem can be reduced to the optimization of the lowpass filter impulse response \( g(k T_S) \) and the bandpass filter impulse response \( g_{Y_{	ext{BP}}}(k T_S) \). Moreover, proceeding this way has two distinct advantages:

- The number of parameters to be optimized is minimum.
- The optimization can be carried out by means of continuous optimization procedures. These are generally less time-consuming than procedures in the discrete parameter space.

When computing the SEs for different implementation schemes of a modem transmitter, the quantization effects have to be considered corresponding to the individual round-off operations. For the structure according to Fig. 2a, for instance, the complex terms \( c_i \exp(-j \phi_i) \) and the orthogonal base \( \{u_1, u_2\} \) are explicitly given in quantized form, whereas for the implementation according to Fig. 2b the SEs have to be quantized only in their final form.

4 Solution to the optimization problem

In order to solve the optimization problem, the requirements posed on the quantised transmitter output sequence \( [x(k T_S)]_Q \) or the SEs, respectively, have to be set up. With knowledge of these requirements, an objective function can be defined. Starting with an appropriate set of initial parameters, the minimization of this objective function yields the desired solution. Note that the optimization procedure is related to the continuous parameter space even if the modem transmitter is to be implemented by digital circuitry. Bearing in mind the symmetry relations of the signal space as outlined in section 3, it is sufficient to restrict the design procedure on the subset \( B_1 \) of all \( M \) SEs.

4.1 Discussion of the requirements

For data transmission over band-limited channels, requirements relating to the frequency and to the time domain have to be considered. On the basis of the CCITT Recommendations for modem transmitters [17], generally the following typical requirements with regard to the frequency domain exist:

1. The magnitude of the spectrum of the quantised transmitter output sequence \( [x(k T_S)]_Q \) has to approximate a bandpass characteristic, which is derived from the lowpass transfer function \(^2\)

\[
G(e^{j2\pi f T_S}) = \begin{cases} 
1, & 0 \leq |f| \leq f_N/(1 - r), \\
\sqrt{0.5} \{1 - \sin [\pi (f - f_N)]/f_N\}, & (1 - r) f_N \leq |f| \leq f_N (1 + r), \\
0, & (1 + r) f_N \leq |f| \leq f_S/2.
\end{cases}
\]

This is achieved by centering eq. (13) at the carrier frequency \( f_c \). Here, \( f_N = 1/(2 T) \) is the Nyquist frequency and \( r \) the roll-off factor by which the slope of the transition band is specified [13].

2. In the usable frequency range \( f_c - f_N \leq |f| \leq f_c + f_N \) the phase response should be essentially linear. A requirement related to the time domain must ensure that

3. the desired signals of the signal space are represented by the quantised SEs both in amplitude and phase as accurately as possible.

As a result of these three requirements, the SEs are defined except for a scaling factor, which is specified by:

4. Scale the SEs such that the available range of numbers between the actual overflow limits is completely utilized by the quantised transmitter output sequence \( [x(k T_S)]_Q \).

First, requirement 3 must be investigated more closely. To this end, we start from the equivalent baseband representation of a modem transmitter of arbitrary structure, as shown in Fig. 4. The derivation of this model is outlined in Appendix B.

The deviations of the quantised SEs from the ideal signals, as qualitatively specified by requirement 3, are mainly caused by two nonlinear effects, namely by

- the quantization of the samples and finite wordlength effects in signal processing, and by
- foldover or aliasing effects. This is due to the fact that the complex carrier exp\((-j \omega_c k T_S)\) is modulated by the only approximately band-limited FIR lowpass spectrum of the impulse response \( g(k T_S) \) (cf. eqs. (1)-(3), (6)).

\(^2\) Subsequently, the modified raised-cosine bandpass characteristic derived from eq. (13) is abbreviated by \( \sqrt{R C (\alpha)} \), where the argument \( \alpha \) represents the roll-off factor.
The foldover and quantization effects are tractable in the time domain. Generally, they cause the spectra of the quantized SEs to lose their symmetry about the carrier frequency \( f_c \). This leads to crosstalk between the quadrature channels, which can be represented by the baseband impulse responses \( q_l \) (\( k T_S \)), as depicted in Fig. 4. Note that the impact of these effects on each SE of the subset \( E_1 \) may be different due to the inherent carrier phases. As a consequence, the equivalent baseband impulse responses \( p_l \) (\( k T_S \)), and \( q_l \) (\( k T_S \)) at a given time \( t \) depend on the actual symbols \( c_l = a_l + j b_l \) to be transmitted. Furthermore, it must be considered that the impulse responses \( g \) (\( k T_S \)) and thus both \( p_l \) (\( k T_S \)), and \( q_l \) (\( k T_S \)) are generally of longer duration than the symbol interval \( T \). Hence, different pulse forms overlap. Obviously, a linear equalizer in the data receiver cannot compensate this kind of nonlinear distortion. On the other hand, requirement 3 can be weakened if the data receiver comprises a linear equalizer. Then, crosstalk may be tolerated by requiring the equivalent impulse responses to lose their symmetry about the carrier. As a consequence, the equivalent transmitter is obtained by multiplying the modulation matrix \( M = \text{diag} \left( e^{j\omega_c k(T-N/2)} T_S, \ldots, e^{-j\omega_c k(T-N/2)} T_S \right) \) by the parameter vector \( g \), which is related to \( h \) by eqs. (15) and (16):

\[
 y = M g .
\]

Finally, the SEs \( \epsilon E_1 \)

\[
 s_\mu = \text{Re} \{ \gamma_\mu y \} , \quad \mu = 1, 2, \ldots, m ,
\]

result which have not been quantized so far. \( \gamma_\mu \) is defined by eq. (5).

In the next step, the SEs \( s_\mu \in E_1 \) have to be scaled according to requirement 4 before quantization. To this end, all possible combinations of every \( n \) SEs \( E_1 \) may be superimposed at any given time can be computed according to eq. (3), because \( \{ \gamma_\mu | \mu = 1, 2, \ldots, M \} \) is a finite set of complex numbers (cf. Fig. 2b). In this procedure the symmetry properties of the signal space, as described in section 3, may advantageously be exploited. Thus, the maximum magnitude of the unscaled transmitter output sequence \( x \) (\( k T_S \)) is obtained. Hence, with the prescribed overflow limitations the scaling factor can be determined such that the available range of numbers is completely used.

Finally, the quantised SEs \( E_1 \) for an implementation according to Fig. 2a using eqs. (5) and (20) result in

\[
 [s_{\mu s}]_Q = \text{Re} \left\{ \left[ \left( c_{\lambda} e^{-j\omega_c k T} \right)_{\lambda} \right]_Q \cdot \left[ u_{s} \right]_Q \right\} , \quad \mu = 1, 2, \ldots, m ,
\]

where scaled quantities are denoted by the subscript \( s \). The square brackets specify the individual operations of quantization. In contrast, the quantization scheme of the structure according to Fig. 2b is quite simple:

\[
 [s_{\mu s}]_Q = \text{Re} \left\{ c_{\lambda} e^{-j\omega_c k T} u_{s} \right\} , \quad \mu = 1, 2, \ldots, m ,
\]

Next, the time domain requirement 3 or 3A, respectively, for the quantised SEs \( E_1 \) is expressed mathematically, since this leads to a simplified definition of the objective function with respect to the frequency domain specifications. The following conclusions are based on the part of the equivalent transmitter baseband model depicted in Fig. 5. Here, the quantized SEs \( [s_{\mu s}]_Q \in E_1 \) are represented by the unquantized (noncausal) impulse responses \( p_{\mu} \) (\( k T_S \)) and \( q_{\mu} \) (\( k T_S \)) of duration \( N T_S \). For the derivation of the equivalent baseband representation according to Fig. 5, the inherent phase position of each of the \( m \) SEs \( [s_{\mu s}]_Q \in E_1 \) are assumed to comply with the desired phase angles in the signal space, as it is, for instance, shown in Fig. 3c.
Fig. 5. Equivalent baseband representation of a modem transmitter used to derive the time-domain requirement: \( \delta_0(t) = \text{unit pulse; } \mu = 1, 2, \ldots, m \)

(c.f. App. B). Therefore, the SEs exhibit the desired phase position if and only if the condition

\[
\arctan \frac{b_{\mu}^{\text{out}}(k T_S)}{a_{\mu}^{\text{out}}(k T_S)} = \arctan \frac{b_{\mu}^{\text{in}}}{a_{\mu}^{\text{in}}}, \quad \mu = 1, 2, \ldots, m, \quad k = -(N-1)/2, \ldots, (N-1)/2, \quad (22)
\]

is met for all \( N \) samples of any symbol \( c_{\mu} \) corresponding to \( \delta_{\mu} \in E_1 \). The notation is obvious from Fig. 5.

In addition, the desired magnitudes \( |c_{\mu}^{\text{out}}(t)|, \forall \mu \) at the output of the (noncausal) baseband pulse shaping filters shall, at the reference time \( t = 0 \), be equal to the corresponding magnitudes \( |c_{\mu}^{\text{in}}| \) of the symbols to be transmitted with the exception of a scaling factor. Therefore, the \( m \) SEs \( \{\delta_{\mu}^{BQ}\} \) may be required to meet the additional conditions:

\[
|c_{\mu}^{\text{out}}(t = 0)| = |c_{\mu}^{\text{in}}|, \quad \mu = 2, \ldots, m. \quad (23)
\]

Inserting eq. (25) the phase condition (22), after a simple rearrangement, results in the squared error function:

\[
e_{\mu} = \sum_{\mu=1}^{m} \frac{(N-1)^2}{(N-1)/2} q_{\mu}^2(k T_S). \quad (26)
\]

Correspondingly, a squared error function can be set up for the SEs' magnitudes according to eq. (24) by again using eq. (25):

\[
e_{M} = \sum_{\mu=2}^{m} \frac{(N-1)^2}{(N-1)/2} \left( \sqrt{p_{\mu}^2(k T_S) + q_{\mu}^2(k T_S)} - \sqrt{p_{\mu}^2(k T_S) + q_{\mu}^2(k T_S)} \right)^2. \quad (27)
\]

Combining the two error functions (26) and (27) in vector notation and considering that, as a consequence of eq. (26), generally \( q_{\mu}^2(k T_S) \ll p_{\mu}^2(k T_S) \) holds, we obtain the ultimate form of the objective function with respect to the time domain requirement 3 according to:

\[
e_{t} = \sum_{\mu=1}^{m} (p_{\mu} - p_{T})^T (p_{\mu} - p_{T}) + q_{\mu}^T q_{\mu}. \quad (28)
\]

Here, the vectors \( p_{\mu} \) and \( q_{\mu} \) are of the same structure as the vector \( g \) according to eq. (14).

When tolerating crosstalk, as discussed in section 4.1, eq. (28) can straightforwardly be reformulated such that a squared error function for the modified time domain requirement 3A results:

\[
e_{t}^A = \sum_{\mu=1}^{m} (p_{\mu} - p_{T})^T (p_{\mu} - p_{T}) + (q_{\mu} - q_{T})^T (q_{\mu} - q_{T}). \quad (29)
\]

As compared to eq. (23), condition (24) does not imply additional computational effort, since the complete \( N \)-point equivalent baseband representation of the quantised SEs must be computed for eq. (22) in any case.

Without loss of generality, for the filter optimization using eqs. (23) and (24) \( c_{\mu}^{\text{in}} \), \( \forall \mu \) may be set to \( c_{\mu}^{\text{in}} = c_{\mu}^{\text{in}} = 1 \). Then the baseband output sequence is given by

\[
c_{\mu}^{\text{out}}(k T_S) = p_{\mu}(k T_S) + j q_{\mu}(k T_S). \quad (25)
\]
Parameter vector \( h \)
\[ \text{dim} \ h = \langle N/2 \rangle \]
compute
\[ g_\mu \in E_1 \]
\[ \mu = 1, 2, \ldots, m \]
scale
\[ g_\mu \in E_1 : \mathbb{Z}_{\mu} \]
\[ \mu = 1, 2, \ldots, m \]
quantise
\[ g_\mu \in E_1 : \mathbb{Z}_{\mu} \]
\[ \mu = 1, 2, \ldots, m \]
calculate MSE \( e_1(s_{\mu}) \) in the frequency domain for arbitrary
\[ \mu \in \{1, 2, \ldots, m\} \]
calculate MSE \( e_1(W) \) in the time domain using the equivalent baseband
representation of \( [s_{\mu}]_0 \)
for \( \mu = 1, 2, \ldots, m \)
compute objective function
\[ e(A) = e_t + \eta e_1(A) \]

Fig. 6. Flow chart of design algorithm

The SE's subscript \( \mu \), which can be selected arbitrarily, is
set to \( \mu = 1 \) in eq. (30). Using the Z-transform, the spectrum of the quantised SE \( [s_{11}]_Q \) is given by
\[ S_{1Q}(e^{j2\pi f/f_S}) = \mathbb{Z} \{ [s_{1k} (kT_S)]_0 \} \mid_{z = e^{j2\pi f/f_S}}. \]  

Furthermore, in eq. (30) \( W(\cdot) \) is a weighting function
accounting for different tolerance requirements and
\( D(\cdot) = \sqrt{RC(\cdot)} \) is the desired magnitude response of the
SEs' spectra to be approximated at the frequencies \( f_k, \ k = 1, 2, \ldots, K \). In the case of particularly stringent spectral requirements in certain narrow frequency bands
(e.g. for the possibility of introducing a backward channel),
the frequency domain squared error function according to
eq (30) may, in these frequency ranges, be extended to
all \( m \) SEs \( [s_{\mu}]_Q \in E_1 \).

Following the above arguments, the overall objective function is now defined by
\[ e(A) = e_t + \eta e_1(A), \]  
where \( \eta \) is a weighting factor \(^5\). The minimization of \( e(t) \)
leads to the optimum equivalent baseband representation
(cf. Fig. 5), from which the desired quantised SEs can be
derived by applying eq. (21) appropriately.

Finally, Fig. 6 provides a comprehensive survey of the
determination of the objective function \( \epsilon = \epsilon (h) \). The linear
phase requirement 2 is inherently met due to the choice of
a symmetrical impulse response of the FIR baseband pulse
shaping filters \( g \) according to eq. (15). Furthermore, the
scaling operation of the SEs is carried out at each call of
the objective function, thus always ensuring requirement 4
being met. Squared error functions have to be computed
only for the frequency and time domain requirements 1
and 3. It is essential to note that the minimization of the
weighted sum of these two terms according to eq. (32)
can be carried out using numerical optimization procedures
related to the continuous parameter space of the equivalent
baseband representation of the modem transmitter. This is
valid regardless of whether the signals in the data trans-
mittmer are processed in a time-discrete or digital manner.

4.3 Determination of a set of initial parameters

Generally, it may be expected that an objective function
made up of more than one criterion such as eq. (32) results
in an ill-conditioned optimization problem. A major part
of the associated difficulties can, however, be overcome by a
suitable choice of the initial set of the FIR lowpass impulse
response samples \( g \).

For the determination of an appropriate set of initial
parameters computer programs, such as CONRIL [18]
and that of McClellan et al. [19] are available, which can be
used to design linear phase FIR filters with arbitrary con-
straints. However, when specifying the requirements for
the initial vectors \( g \) or \( h \), respectively, two aspects have to be
considered thoroughly:

1. Due to the process of modulating a carrier of frequency
\( f_c \) by the FIR lowpass impulse response \( h \) spectral fold-
over arises. In order to control these effects, \( h \) must be
somewhat overspecified as compared to the bandpass
SEs particularly in the vicinity about twice the carrier
frequency, which is folded onto the usable spectrum of the
bandpass SEs by the modulation process.

2. Additional distortions of the baseband spectra arise from
quantization of the samples of the SEs. In order to con-
trol these effects, a further overspecification of \( h \) is
necessary which, in contrast to foldover, can essentially
be restricted to frequency bands with stringent tolerance
requirements such as high attenuation (e.g. for a back-
ward channel) or very small passband ripple. For details
refer to [20], for instance.

5 Example

As an example, a pseudo 8-phase DPSK modem transmitter
was designed on the basis of the CCITT Recommendation
V.26 bis [17], which deals with the transmission of data at
bit rate of 2400 bits per second switched telephone net-
work. Since, in this case, \( f_c = 1800 \text{ Hz} \) and \( 1/T = 1200 \text{ baud} \),
for the symbol rotation angle \( \varphi_i \) according to eq. (4) results:
\( \varphi = \omega_i T i = 3 \pi i \). Thus, each symbol \( c_{1j} + j c_{1j} \) is trans-
mformed to \( y_i = c_{1j} + j c_{1j} = -c_{1j} \) by rotation, which belongs
\(^5\) Superscript A is used to distinguish the two modifications
of the time domain requirement 3.

126
itself to the set of symbols \( \{ c_1, \ldots, c_8 \} \) to be transmitted. As a consequence, the set \( E \) just comprises \( M = 8 \) SEs. Hence, in order to exploit the symmetry properties of the signal space as outlined in section 3, we start with an arrangement of these eight SEs in the phase plane as shown in Fig. 3c, of which only two (here, for example, \( z_1 \) and \( z_2 \)) have to be included in the design algorithm.

Under these conditions an implementation of the transmitter according to Fig. 2b is most efficient, for which reason the design has been carried out for this structure. Requirements concerning linear phase are specified by CCITT Rec. V. 26 bis, whereas no specifications are included with respect to the desired magnitude response of the transmitter spectrum. However, in order to comply with requirements due to the Federal German Telecommunications Administration, the design was based on a desired function \( D(\cdot) = \sqrt{R C(\cdot)} \) with a roll-off factor of \( r \approx 0.85 \), where the weighting function \( W(\cdot) \) was chosen according to the tolerance constraints. Furthermore, within a narrow frequency band about 420 Hz, a spectral purity of approximately 60 dB is required for an FSK backward channel (symbol rate up to 75 baud).

For signal processing, a sampling frequency of \( f_s = 7.2 \text{ kHz} = 6/T \text{ Baud} \) was chosen. Moreover, at any given time \( n = 3 \) samples of SEs staggered in time could be superimposed to form the transmitter output sequence \( x(kT_3) \) according to eq. (3). Hence, the SEs are of length \( N = 18 \).

When designing the initial FIR lowpass impulse response \( g \) or \( h \), respectively, applying [19], an attenuation of 75 dB was specified for the narrow frequency bands about \( f_c \pm 420 \text{ Hz} \), which are folded onto the backward channel by the modulation process. In order to appropriately constrain the foldover effects on the main usable spectrum, a minimum attenuation of 45 dB–50 dB was required in the frequency band about twice the carrier frequency.

Starting with the initial vector \( h_0 \), obtained as outlined above, the design of the modem transmitter was carried out using the procedure as described in the previous section 4. The objective function was minimized by an optimization algorithm according to Powell [21], modified by Müller [22]. To this end, the Powell procedure was combined with a random-search as proposed by Bandler et al. [23]. As a design result, Fig. 7 shows the two SEs \( [s_{18}]_Q \) and \( [s_{23}]_Q \) of

---

Fig. 7. SEs \([s_{18}]_Q \) and \([s_{23}]_Q \) in the time domain, designed for a modem transmitter according to CCITT Rec. V. 26 bis [17]
the subset $E_1$, which are scaled and quantized with 8 bits (quantization step size $Q = 2^{-7}$). In a modern transmitter implementation according to Fig. 2b these SEs are stored in the SEs' memory. On the average, corresponding impulse responses of the equivalent baseband representations of the quantized SEs differ from each other by $0.25\, Q$, the maximum difference being $1.59\, Q$.

Fig. 8. shows the magnitude response of the spectrum $S_{1Q}(\exp(j2\pi ffS))$ and the difference between $|S_{1Q}()|\, |S_{2Q}()|$ (enlarged scale). It is obvious that the deviations of the magnitude responses of the spectra from each other are very small in a wide frequency range about the carrier frequency. Furthermore, Fig. 8 shows that the requirements concerning the 75 Baud FSK backward channel at 420 Hz are met. The attenuation of spectral components in the frequency ranges $0 \leq f \leq 600$ Hz and $3\, kHz \leq f \leq f_s/2$ is essentially higher than $30\, dB$ as required. Finally, Fig. 9 shows that the linear phase requirement is met with extreme accuracy if it is compared with the allowable tolerance according to [17].
Conclusions

In this paper an algorithm was presented which is suitable for the design of time-discrete and digital modem transmitters of any arbitrary structure under the restriction of linear modulation forms (SE) of finite duration. In order to generate a data transmitter output sequence, replicas of the SEs, which are staggered in time with respect to each other by integer multiples of the symbol interval $T$, have to be superimposed.

Regardless of the particular implementation of the modem transmitter, these SEs can always be derived from a non-quantized lowpass impulse response (in companion with a non-quantized bandpass impulse response, if VSB-modulation is applied) of finite duration, whose samples represent the parameters of the objective function to be minimized.

In the sequel, the main features characterizing the application of this design procedure to QAM data transmitters with digital signal processing are summarized:

1. The set of parameters $g$ consists of non-quantized samples of a symmetrical (linear phase) lowpass impulse response of finite duration.

2. Minimization of the objective function $e^{(A)}$ is performed in the continuous parameter space. Hence, it is generally less time-consuming than an optimization with discrete parameters. A suitable starting vector $\hat{h}$ has to be supplied, as it is commonplace to most optimization problems.

3. Throughout the design procedure the symmetry of the lowpass impulse response is always maintained. As a consequence, the SEs as well as the quantized transmitter output sequence exhibit extremely good linear phase properties.

4. Only a subset of the bandpass SEs $S_{g_k}$ has to be included in the design algorithm. The SEs $S_{g_k} \in E_2 (\mu = 1, 2, \ldots, m)$ for direct inclusion in the design procedure are selected by utilizing the symmetry properties of the signal space. Note that all SEs $S_{g_k} \in E_2 (\mu = m + 1, m + 2, \ldots, M)$ can be derived from the subset $E_1 (E = E_1 \cup E_2)$ without arithmetic operations and thus free of errors.

5. At the beginning of each call of the objective function the SEs $S_{g_k} \in E_1$ are scaled anew. Thus, the range of numbers within the overflow limitations can completely be exploited.

6. The time-domain objective function $e^{(A)}$ applied to the equivalent baseband representation of the quantized SEs $[\hat{g}_{\mu}]_Q \in E_1$ implies the following requirements:

   - Minimization of the magnitude deviations of the spectra of the SEs $[\hat{g}_{\mu}]_Q \in E$ from each other.
   - Minimization of the nonlinearities due to quantization.
   - Minimization of spectral foldover effects due to the modulation process applying an approximately band-limited FIR lowpass signal $\hat{h}$.

7. With the objective function $e_1$, the frequency domain requirement is controlled for only one quantized SE of the subset $E_1$.

8. The objective function $e^{(A)} = e_1 + \eta e^{(A)}$ to be minimized is composed of the weighted sum of the squared error norms of the frequency and time domain requirements.

The efficiency of this design procedure has been demonstrated by an example of a data transmitter according to CCITT Rec. V.26 bis.

Acknowledgement

The author is greatly indebted to Dr. Till and Dr. Wrezinskiy as well as to Dr. Schenk and Dr. Konnwyer (formerly at the Institut für Nachrichtentechnik, Universität Erlangen-Nürnberg) for their readiness to stimulating discussions on the topic of this paper. Furthermore, the author would like to express his gratitude to Dr. Greismer for his constructive criticism on the initial manuscript as well as to Mr. Story and Mr. Hinsenkamp for their valuable support in the establishment of the English version of this paper.

Appendix A

The relationship between the SE vectors $s_2$ and $s_8$, which is defined by eq. (9) using the permutation matrix $P$ according to eq. (10), may be reformulated as

$$s_8 (k T_8) = s_2 (-k T_8) ,$$  \hspace{2cm} (A1)

where $k$ ranges from $-(N - 1)/2$ to $(N - 1)/2$ with a unity step size. If the two real signals of eq. (A1) are subjected to the two-sided Z-transform, we obtain for the left-hand side

$$Z \{ s_8 (k T_8) \} = \sum_{k=-\infty}^{\infty} s_8 (k T_8) z^{-k} = S_8 (z) ,$$  \hspace{2cm} (A2)

and for the right-hand side [24]:

$$Z \{ s_2 (-k T_8) \} = \sum_{k=-\infty}^{\infty} s_2 (-k T_8) z^{-k}$$

$$= \sum_{k=-\infty}^{\infty} s_2 (k T_8) z^{k} = S_2 (z^{-1}) .$$  \hspace{2cm} (A3)

Since the spectra of real signals exhibit Hermitian symmetry, according to eq. (A1) it turns out for real frequencies:

$$S_8 (e^{j2 \pi f T_8}) = |S_8 (e^{-j2 \pi f T_8})| = |S_2 (e^{j2 \pi f T_8})|,$$  \hspace{2cm} (A4)

$$\Re \{ S_8 (e^{j2 \pi f T_8}) \} = \Re \{ S_2 (e^{-j2 \pi f T_8}) \} = - \Re \{ S_2 (e^{j2 \pi f T_8}) \}.$$  \hspace{2cm} (A5)

If the angle $\psi_2 = \psi = \pi/4$ is represented by the SE $s_2$ in the signal space according to Fig. 3b, then it is obvious from eq. (A5) that $s_8$ represents $\psi_8 = - \psi_2 = - \pi/4$ as required. Moreover, because of eq. (A4) the magnitudes of both spectra are identical. Q. E. D.

Note that this proof is valid for arbitrary phase positions and for quantised signals as well.

Appendix B

The derivation of the equivalent baseband representation

$$[P] = \begin{bmatrix} p_{-(N-1)/2} & \cdots & p_{(N-1)/2} \\ q_{-(N-1)/2} & \cdots & q_{(N-1)/2} \end{bmatrix}^T , \quad \mu = 1, 2, \ldots, m ,$$  \hspace{2cm} (B1)

ntzArchiv Bd. 6 (1984) H. 6

129
Decomposing \((M \cdot R_{\mu})^{-1}\) into real and imaginary parts according to
\[
(M \cdot R_{\mu})^{-1} = C_{\mu} + j S_{\mu}, \quad \forall \mu,
\] (B7)
where
\[
C_{\mu} = \text{diag} \left( \cos \left( -\omega_{c} \left( (N - 1)/2 \right) T_{S} - \delta_{\mu} \right), \ldots, \sin \left( \omega_{c} \left( (N - 1)/2 \right) T_{S} - \delta_{\mu} \right) \right), \quad \forall \mu,
\] (B8)
and
\[
S_{\mu} = \text{diag} \left( \sin \left( -\omega_{c} \left( (N - 1)/2 \right) T_{S} - \delta_{\mu} \right), \ldots, \right)
\] (B9)
and introducing eq. (B2) into eq. (B6) finally leads to
\[
\begin{pmatrix} p_{\mu} \\ q_{\mu} \end{pmatrix} = \begin{pmatrix} C_{\mu} - S_{\mu} \\ S_{\mu} C_{\mu} \end{pmatrix} \begin{pmatrix} \delta_{\mu} \\ \delta_{\mu} \end{pmatrix}, \quad \forall \mu. \tag{B10}
\]

The computation of the Hilbert transforms of the SEs to be inserted into eq. (B10) is elementary, if the transformations are carried out in the frequency domain. Here, the Hilbert transform is represented by the time-discrete transfer function
\[
H \left( e^{j2\pi f/2} \right) = \begin{cases} -j \text{sign} (f), & |f| < f_{S}/2, \\ 0, & |f| = f_{S}/2. \end{cases} \tag{B11}
\]

Thus, we obtain the analytical SEs in the frequency domain in two steps: a) Subject the SEs to the discrete Fourier transform (DFT), which provides exact results due to the finite duration of the SEs. b) Relate the transfer function \(H(\cdot)\) according to eq. (B11) to the spectra of the SEs in compliance with eq. (B2). By means of the inverse discrete Fourier transform (IDFT), the analytical SEs are finally obtained in the time domain:
\[
\delta_{\mu} = \text{IDFT} \left\{ \left[ 1 + j H \left( e^{j2\pi f/2} \right) \right] \text{DFT} \left\{ \delta_{\mu} \right\} \right\}, \quad \forall \mu. \tag{B12}
\]

References

[9] Snijders, F. A. M.: Microprocessor implementation of data modems. Eurocon '77, Venice (Italy), May 3—7, 1977, pp. 2.10.3.1—2.10.3.7


[17] CCITT Orange Book VIII: Data Transmission over the Telephone Network


